ISO DRAFT CHANGES
DECEMBER 1985

This earlier (10th) draft I believe is better than the final (12th) draft dated 1987

FLUID FLOW
MEASUREMENT UNCERTAINTY

DRAFT

REVISION OF ISO/DIS 5168
FOR

INTERNATIONAL ORGANIZATION
FOR STANDARDIZATION COMMITTEE
ISO TC30 SC9

UNITED
TECHNOLOGIES
PRATT&WHITNEY

Paul – This is the best standard I ever wrote on measurement uncertainty but I had to put in changes I opposed in the later two drafts to get the French vote.

You may say it was approved by the ISO TC30 SC9 committee and world vote but never published by the French secretariat of TC30.

Contents

1 Scope and field of application

Whenever a measurement of flowrate (discharge) is made the value obtained from the experimental data is simply the best possible estimate of the true flowrate. In practice, the true flowrate may be slightly greater or less than this value.

This International Standard details step by step procedures for the evaluation of uncertainties in individual flow measurements arising from both random and systematic errors and for the propagation of these errors into the uncertainty of the test results. These procedures enable the following processes to be effected:

a) Estimation of the accuracy of the test results derived from flowrate measurement

b) Selection of a proper measuring method and devices to achieve a required level of accuracy of flowrate measurement

c) Comparison of the results of measurement

d) Control over the sources of errors contributing to a total uncertainty

e) Refinement of the results of measurement as data accumulate

NOTE. It is assumed that the measurement process is carefully controlled and that all calibration corrections have been applied.

This standard describes the calculations required in order to arrive at an estimate of the interval within which the true value of the flowrate may be expected to lie. The principle of these calculations is applicable to any flow measurement method, whether the flow is in open channel or in closed conduit. Although this standard has been drafted taking mainly into account the sources of error due to the instrumentation, it shall be emphasized that the errors due to the flow itself (velocity distribution, turbulence, etc...) and to its effect on the method and on the response of the instrument can be of great importance with certain methods of flow measurement (see 5.7). Where a particular device or technique is used, some simplifications may be possible or special reference may have to be made to specific sources of error not identified in this Standard. Therefore reference should be made to the "Uncertainty of measurement" clause of the appropriate International Standard dealing with that device or technique.

2 References

ISO 748 Liquid flow measurement in open channels — velocity area methods

ISO 772 Liquid flow measurement in open channels — vocabulary and symbols

ISO 3534 Statistics — vocabulary and symbols (1977)

ISO 4006 Measurement of fluid flow in closed conduits — vocabulary and symbols

ISO 4360 Liquid flow measurement in open channels by weirs and flumes — triangular profile weirs

ISO 5725 Precision of test methods — determination of repeatability and reproducibility by inter-laboratory tests

3 Glossary and Notation

3.1 Notation

β — The systematic error, the fixed, or constant component of the total error, δ.

δ — The total error.

ε — Random (precision) error

B — The estimate of the upper and lower limit of the symmetrical systematic error, β.

$$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^{\,2}}$$

B^+, B^- — The upper and lower limits of a non-symmetrical systematic error.

B_{ij} — An estimate of the upper limit of an elemental systematic error. The j subscript indicates the process, i.e.

j = 1 calibration error,
= 2 data acquisition
= 3 data reduction

The i subscript is the number assigned to a given elemental source of error. If i is more than a single digit, the comma is used between i and j.

bar ($\bar{\ }$) The mean value of a variable.

C The number of coefficients estimated in regression analysis

Δ The difference between measurements

K Calibration constant

M Number of redundant instruments or tests

N Sample size

$\hat{r}$ The sample correlation is an estimate of the true, unknown population correlation coefficient, ρ.

σ^2 The variance, the square of the standard deviation

μ Population mean.

S^2 An unbiased estimate of the variance, σ^2.

$S_{\Delta r}$ An estimate of the experimental standard deviation of $\Delta r = r_1 - r_2$

$$= \sqrt{s_{r_1}^2 + s_{r_2}^2}$$

S_{ij} The estimate of the experimental standard deviation from one elemental source. The subscripts are the same as the elemental systematic error limits in the foregoing.

$$S = \sqrt{\sum_i \sum_j S_{ij}{}^2}$$

S_y Estimate of the experimental standard deviation of the variable Y

S_{pooled}

$$= \sqrt{\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (X_{ij} - \bar{X}_i)^2}{M(N-1)}}$$

t_{95} Student's statistical parameter at the 95 percent confidence level. The degrees of freedom, ν, of the sample estimate of the standard deviation is needed to obtain the t value.

U^+, U^- The upper and lower limits of a non-symmetrical uncertainty interval.

U_{99}
U_{ADD} $= B + t_{95} S_{\bar{x}}$, provides ~ 99% coverage.

U_{95}
U_{RSS} $= \sqrt{B^2 + (t_{95} S_{\bar{x}})^2}$, provides ~ 95% coverage.

$\bar{x}$ Arithmetic mean of the data values; x_i

$\bar{x}$ Sample average of measurements

$$\bar{x} = \frac{\sum_{i=1}^{N} X_i}{N}$$

x_i The value of x at the i-th data point

x_j The j-th independent variable (in multiple linear regression)

x_{ji} The value of x_j at the i-th data point

y' The value of y predicted by the equation of the fitted curve.

$\bar{Y}$ Arithmetic mean of the n measurements of the variable Y.

y_i The value of y at the i-th data point

Subscripts

i The number of the error source within the error category; also, a general index.

ADD The additive model

RSS The root-sum-square model

NOTE — These statistical symbols are in accordance with ISO3534 Statistics — Vocabulary and Symbols

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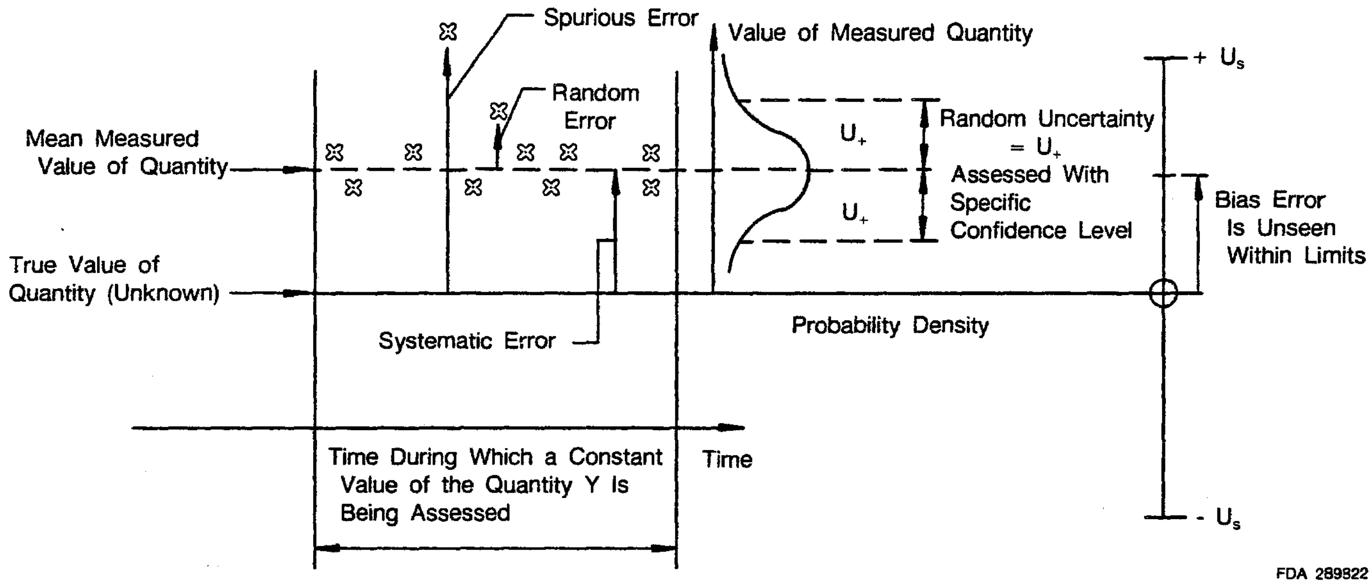


Figure 1 —

3.2 Glossary

3.2.1 bias — see 3.2.36 and figure 2.

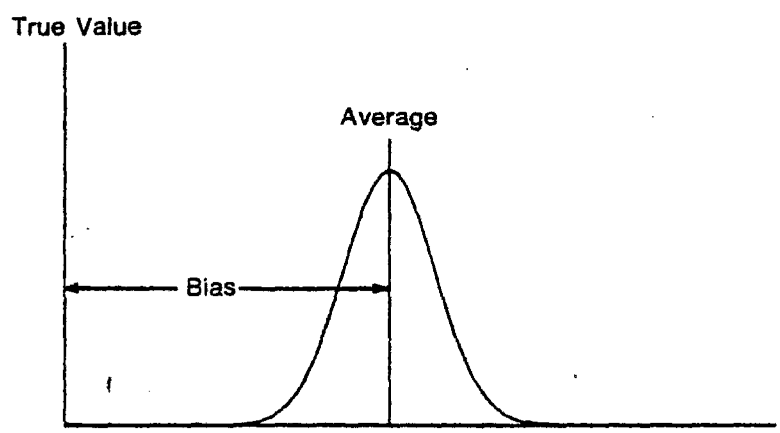


Figure 2 — Bias

3.2.2 *bias limit*

The estimate of the upper limit of the bias (systematic) error.

3.2.3 *calibration curve*

The locus of points obtained by plotting some index of the calibration response of a flowmeter against some function of the flowrate.

3.2.4 *confidence interval*

The interval within which the true value is expected to lie, with a specified confidence level.

3.2.5 *correction*

A value which must be added algebraically to the indicated value to obtain the corrected result. It is numerically the same as a known error, but of opposite sign.

3.2.6 *correlation coefficient*

A measure of the linear interdependence between two variables. It varies between -1 and $+1$ with the intermediate value of zero indicating the absence of correlation. The limiting values indicate perfect negative (inverse) or positive correlation (figure 3).

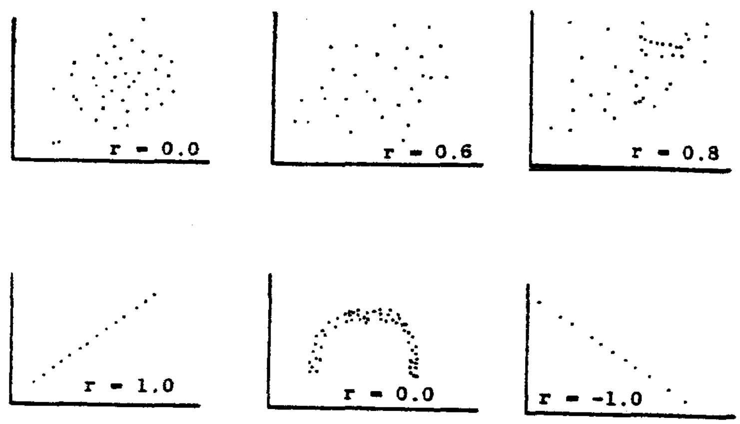


Figure 3 — Correlation Coefficients

3.2.7 *coverage*

The percentage frequency that an interval estimate of a parameter contains the true value. Ninety-five percent confidence intervals provide 95% coverage of the true value. That is, in repeated sampling when a 95% confidence interval is constructed for each sample, over the long run the intervals will contain the true value 95% of the time.

3.2.8 *distribution — see frequency distribution*

3.2.9 *error*

In a result, the difference between the measured and true values of the quantity measured.

3.2.10 *estimate*

A value calculated from a sample of data as a substitute for an unknown population parameter. For example, the experimental standard deviation (S) is the estimate which describes the population standard deviation (σ).

3.2.11 *fossilization*

In the calibration process, live, random errors may become fixed, systematic (fossilized) errors when only a single calibration is relevant.

3.2.12 *influence (sensitivity) coefficient*

The error propagated to the result due to unit error in the measurement. (See 7.4)

3.2.13 laboratory standard

An instrument which is calibrated periodically by the primary test facility. The laboratory standard may also be called a transfer standard.

3.2.14 mean — see average value

3.2.15 measurement error

The collective term meaning the difference between the true value and the measured value. It includes both systematic and random components.

3.2.16 Monte Carlo simulation

A mathematical model of a system with random elements, usually computer adapted, whose outcome depends on the application of randomly generated numbers.

3.2.17 observed value

The value of a characteristic determined as the result of an observation or test.

3.2.18 standard error of the mean

An estimate of the scatter in a set of sample means based on a given sample of size N. Then the standard error of the mean is: $S/\sqrt{N}$

3.2.19 statistical quality control chart

A chart on which limits are drawn and on which are plotted values of any statistic computed from successive samples of a production.

The statistics which are used (mean, range, percent defective, etc.) define the different kinds of control charts.

NOTES: 1) Systematic errors and their causes may be known or unknown.

3.2.20 Taylor's series

A power series to calculate the value of a function at a point in the neighborhood of some reference point. The series expresses the difference or differential between the new point and the reference point in terms of the successive derivatives of the function. Its form is:

$$f(X) - f(a) = \sum_{r=1}^{r=n-1} \frac{(X-a)^r}{r!} f^r(a) + R_n$$

where $f^r(a)$ denotes the value of the r-th derivative of f(x) at the reference point X = a. Commonly, if the series converges, the remainder R_n is made infinitesimal by selecting an artibrary number of terms and usually only the first term is used.

3.2.21 uncertainty

An estimate attached to an observation or a test result which characterizes the range of values within which the true value is asserted to lie. Note: Uncertainty of a measurement comprises, in general, many components. Some of these components may be estimated on the basis of the statistical distribution of the results of a series of measurements and can be characterized by the experimental standard deviation. Estimates of other components can only be based on experience or other information.

3.2.22 Welch-Satterthwaite degrees of freedom

A method for estimating degrees of freedom of the result when combining experimental standard deviations with unequal degrees of freedom.

4 General principles of measurement uncertainty analysis

4.1 Nature of errors

All measurements have errors even after all known corrections and calibrations have been applied. The errors may be positive or negative and may be of a variable magnitude. Many errors vary with time. Some have very short periods while others vary daily, weekly, seasonally or yearly. Those which remain constant or apparently constant during the test are called biases, or systematic errors. The actual errors are rarely known; however, upper bounds on the errors can be estimated. The objective is to construct an uncertainty interval within which the true value will lie.

Errors are the differences between the measurements and the true value which is always unknown. The total measurement error, δ, is divided into two components: β, a fixed systematic error and a random error, ε, as shown in figure 4. In some cases, the true value may be arbitrarily defined as the value that would be obtained by a specific metrology laboratory.

Uncertainty is an estimate of the error which in most cases would not be exceeded. There are three types of error to be considered:

a) random errors — see 4.2

b) systematic (bias) errors — see 4.3

c) spurious errors or blunders (assumed zero) — see 4.4

It is rarely possible to give an absolute upper limit to the value of the error. It is, therefore, more practicable to give an interval within which the true value of the measured quantity can be expected to lie with a suitably high probability. This "uncertainty interval" is shown as $[\overline{X} - U, \overline{X} + U]$ in figure 5 (the interval is twice the calculated uncertainty).

Since measurement systems are subject to two types of errors, systematic and random, it follows that an accurate measurement is one that has both small random and small systematic errors (see figure 6).

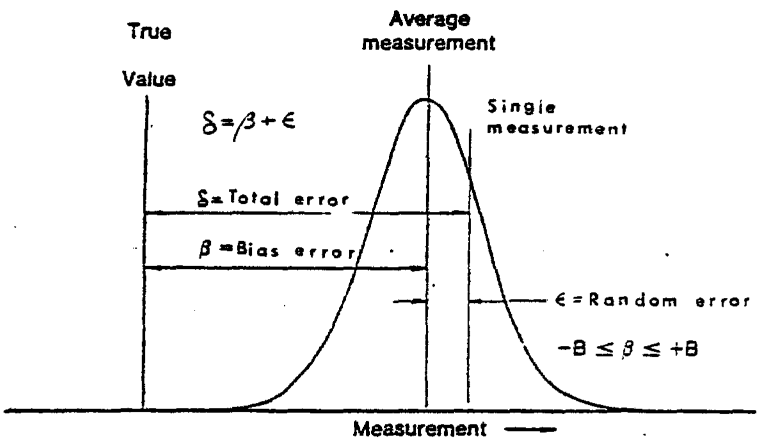


Figure 4 — Measurement error

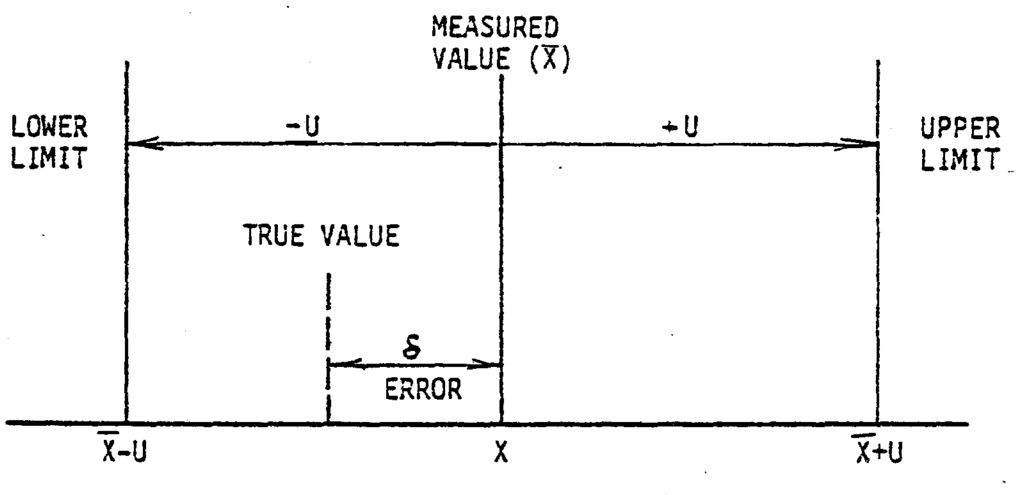


Figure 5 — Uncertainty interval $\overline{X} - U$

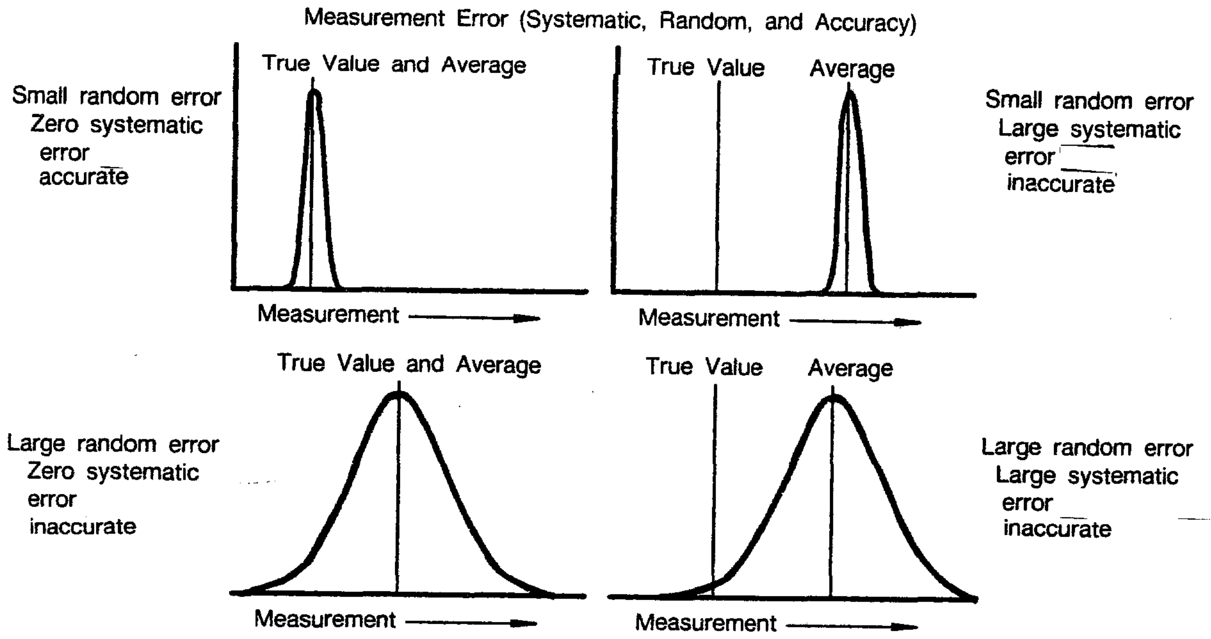


Figure 6 — Measurement error

4.2 Random error (precision)

Random errors are sometimes referred to as precision errors. Random errors are caused by numerous, small, independent influences which prevent a measurement system from delivering the same reading when supplied with the same input value of the quantity being measured. The data points deviate from the mean in accordance with the laws of chance, such that the distribution usually approaches a normal distribution as the number of data points is increased. The variation between repeated measurements is called random or precision error. The standard deviation (σ), figure 7, is used as a measure of the random error, ε. A large standard deviation means large scatter in the measurements. The statistic (S) is calculated from a sample to estimate the standard deviation and is called the experimental standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{N - 1}} \qquad (1)$$

Where

N is the number of measurements and
$\bar{X}$ is the average value of individual measurements, X.

For the normal distribution, the interval $\bar{X} \pm t_{95}\, S/\sqrt{N}$ will include the true mean, μ, approximately 95% of the time. When the sample size is small, it is necessary to use the Student's t values at the 95% level. For sample sizes equal to or greater than 30, two experimental standard deviations (2S) are used as an estimate of the upper limit of random error. This is explained in Annex C.

The random error in the result can be reduced by making as many measurements as possible of the variable and using the arithmetic mean value, since the standard deviation of the mean of N independent measurements is $\sqrt{N}$ times smaller than the standard deviation of the measurements themselves.

$$\sigma_{average} = \frac{\sigma_{individual}}{\sqrt{N}} \qquad (2)$$

and, analogously

$$S_{\bar{x}} = \frac{S}{\sqrt{N}} \qquad (3)$$

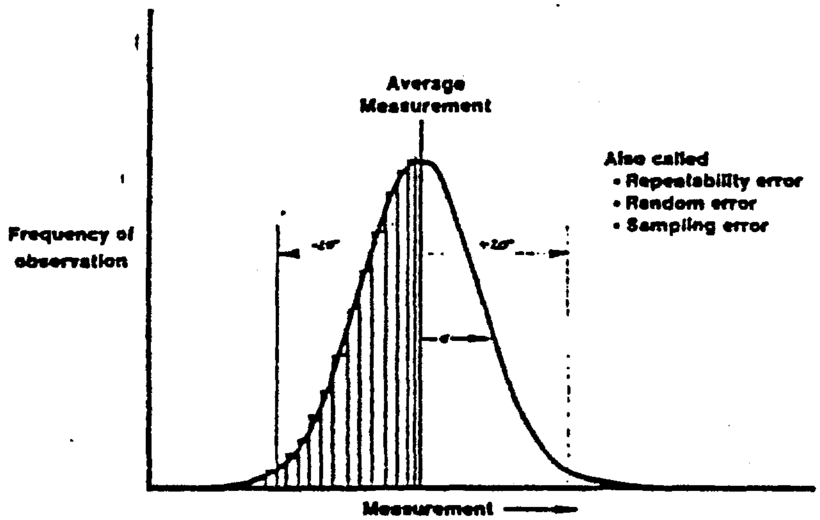


Figure 7 — Random error

4.3 Systematic error (bias)

The second component of the total error is the systematic error, β. At each flow level this error is constant for the duration of the test (figure 4). In repeated measurements of a given sample, each measurement has the same systematic error. The systematic error can be determined only when the measurements are compared with the true value of the quantity measured and this is rarely possible.

The original ISO 5168 had three components of error-random, systematic and systematic that varies with flow level. Within this revision, only the first two components are used to simplify the analysis, recognizing that both components may vary at different levels of flow.

Every effort shall be made to identify and account for all significant systematic errors. These may arise from imperfect (1) calibration corrections, (2) instrumentation installation, and (3) data reduction, and may include (4) human errors and (5) method errors. As the true systematic error is never known, an upper limit, B, is used in the uncertainty analysis.

In most cases, the systematic error, β, is equally likely to be plus or minus about the measurement. That is, it is not known if the systematic error is positive or negative, and the systematic error limit reflects this as ± B. The systematic error limit, B, is estimated as an upper limit of the systematic error, β.

4.4 Spurious errors

These are errors such as human mistakes, or instrument malfunction, which invalidate a measurement; for example, the transposing of numbers in recording data or the presence of pockets of air in leads from a water line to a manometer. Such errors cannot be treated with statistical analysis and the measurement should be discarded. Every effort should be made to eliminate spurious errors to properly control the measurement process.

To ensure control, all measurements should be monitored with statistical quality control charts. Drifts, trends, and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are required for effective control. It is assumed herein that these precautions are observed and that the measurement process is in control; if not, the methods described are invalid.

After all obvious mistakes have been corrected or removed, there may remain a few observations which are suspicious solely because of their magnitude.

For errors of this nature, the statistical outlier tests given in annex D should be used. These tests assume the observations are normally distributed. It is necessary to recalculate the experimental standard deviation of the distribution of observations whenever a datum is discarded as a result of the outlier test. It should also be emphasized that outliers should not be discarded unless there is an independent technical reason for believing that spurious errors may exist: data should not lightly be thrown away.

4.5 Combining elemental errors

The test objective, test duration and the number of calibrations related to the test affect the classification of errors into systematic and random error components. Guidelines will be presented in clause 6.

After all elemental errors have been identified and estimated as calibration, data acquisition, data reduction, methodic errors and subjective errors, a method for combining the elemental random and systematic error limits into the random and systematic error limits of the measurement is needed. The root-sum-square or quadrature combination is recommended.

$$S = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} S_{ij}^2} \tag{4}$$

$$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2} \tag{5}$$

4.6 Uncertainty of measurements

The measurement uncertainity analysis will be completed when:

a) The systematic error limits and standard deviations of the measure have been propagated to errors in the test result, keeping systematic and random errors separate

b) If small samples are involved, an estimate of the degrees of freedom of the experimental standard deviation of the test result has been calculated from the Welch-Satterthwaite formula. (see annex C)

c) The random and systematic errors are combined into a single number to express a reasonable limit for error.

For simplicity of presentation, a single number, U, is needed to express a reasonable limit of error. The single number, some combination of the systematic error and random error limits, must have a simple interpretation (like the largest error reasonably expected), and be useful without complex explanation. For example, the true value of the measurement is expected to lie within the interval

$$[\bar{X} - U, \bar{X} + U]. \tag{6}$$

Since systematic uncertainties are based on judgement and not on data, there is no way of combining systematic and random uncertainties to produce a single uncertainty figure with a statistically rigorous confidence level. However, since it is accepted that a single figure for the uncertainty of a measurement is often required, two alternative methods of combination are permitted.

1) Linear addition:

$$U_{ADD} = B + t_{95} S_{\bar{x}} \tag{7}$$

2) Root-sum-square combination:

$$U_{RSS} = \sqrt{B^2 + (t_{95} S_{\bar{x}})^2} \tag{8}$$

Typically, U_{ADD} will have a coverage of approximately 99 percent, and U_{RSS} will have a coverage of approximately 95 percent. (See Annex F.)

where B is the systematic error limit from equation (5) and $S_{\bar{x}}$ is the experimental standard deviation of the mean (equations (4) and (3)). If large samples ($N > 30$) are used to calculate S, the value 2.0 may be used for t_{95} for simplicity. If small samples ($N \leq 30$) are used to calculate S, the methods in annex C are required. There are three situations where it is possible to develop a statistical confidence interval for the uncertainty interval:

a) If the systematic error limits are based on interlaboratory comparisons, the method is presented in ISO 5725.

b) If the distribution of the systematic error limits are assumed to have a rectangular distribution, the method is shown in annex E.

c) If the systematic error is judged to be negligible compared to the random error, the uncertainty interval is the test result plus and minus t_{95} $S_{\bar{x}}$, which is a 95% confidence interval.

4.7 Propagation of measurement errors to test result errors

If the test result is a function of several measurements, the random error and systematic error limits of the measurements must be combined or propagated to the test result using sensitivity factors, θ, that relate the measurement to the test result. Small sample methods are given in annex C.

In general, for m measurements, the random error and systematic error of the test result is obtained as follows:

$$S_R = \sqrt{\sum_{\text{all } m} (\theta_m S_m)^2} \qquad (9)$$

$$B_R = \sqrt{\sum_{\text{all } m} (\theta_m B_m)^2} \qquad (10)$$

The uncertainty intervals for the test result are formed in the same manner as described for the measurement in 4.6.

4.8 Uncertainty analysis before and after measurement

Uncertainty analysis before measurement allows corrective action to be taken prior to the test to reduce uncertainties when they are too large or when the difference to be detected in the test is the same size or smaller than the predicted uncertainty. Uncertainty analysis before the test can identify the most cost effective corrective action and the most accurate measurement method.

The pretest uncertainty analysis is based on data and information that exists before the test, such as calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analysis, expert opinions and, if necessary, special tests. With complex tests, there are often alternatives to evaluate prior to the test such as different test designs, instrumentation arrangements, alternative calculation procedures and concommitant variables. Corrective action resulting from this pretest analysis may include

a) Improvements to instrumentation if the errors are unacceptable

b) selection of a different measurement or calibration method

c) repeated testing and/or increased sample sizes if the random errors are unacceptably high. The standard error of the mean is reduced as the number of samples used to calculate the mean is increased.

d) Instead of repeated testing the test duration can be extended, in order to average the output scatter (noise) of the flowmeter, resulting in a small random error per observation.

(example — ultrasonic and vortex shedding meters may have to be calibrated against a master meter allowing longer test times than allowed by a micro prover.)

e) Rotating flowmeters usually generate an output showing a periodic cycle superimposed on an average meter factor. In this case the test duration shall be matched to an integer multitude of half or full periodic intervals in order to obtain the shortest test times.

(example — In calibrating positive displacement meters with a small volume micro prover, the double chronometry pulses shall be compared to an integer of pulses generated per revolution of the meter.)

Several iterations may be required in order to obtain the required accuracy.

Posttest analysis is based on the actual measurement data. It is required to establish the final uncertainty intervals. It is also used to confirm the pretest estimates and/or to identify data validity problems. When redundant instrumentation or calculation methods are available, the individual uncertainty intervals should be compared for consistency with each other and with the pretest uncertainty analysis. If the uncertainty intervals do not overlap, a problem is indicated. The posttest random error limits should be compared with the pretest predictions.

5 Identification and classification of elemental measurement errors

5.1 Summary of procedure

Make a complete, exhaustive list of every possible measurement error for all measurements that affect the end test result. For convenience, group them by some or all of the following categories: (1) calibration, (2) data acquisition, (3) data reduction, (4) errors of method and (5) subjective or personal. Within each category, there may be systematic and/or random errors.

5.2 Systematic (bias) vs. random (precision)

Systematic errors are those which remain constant in the process of measurement.

Typical examples of systematic errors of flow-rate measurements are:

a) errors from a single flowmeter calibration

b) errors of determination of the constants in the working formula of a measuring method

c) errors due to truncating instead of rounding off the results of measurement.

Where the value and sign of a systematic error are known, it is assumed to be corrected (the correction being equal in value and opposite in sign to the systematic error). Inaccuracy of the correction results in a residual systematic error.

Random errors are those that produce variation (not predictable) in repeated measurements of the same quantity.

Typical random errors associated with flowrate measurement are those caused by inaccurate reading of the scale of a measuring instrument or by the scatter of the output signal of an instrument.

The effect of random errors may be reduced by averaging multiple results of the same value of the quantity.

The preliminary decision to determine if a given elemental source contributes to systematic error, random error or both, is made by adopting the recommendation: the uncertainty of a measurement should be put into one of two categories depending on how the uncertainty is derived. A random uncertainty is derived by a statistical analysis of repeated measurements while a systematic uncertainty is estimated by nonstatistical methods. This recommendation avoids a complex decision and keeps the statistical estimates separate from the judgement estimates as long as possible. The decision is preliminary and will be reviewed after consideration of the defined measurement process.

5.3 Measurement error categorization

Possible error sources can be divided arbitrarily into three to five categories:

1) Calibration Errors (see 5.4)
2) Data Acquisition Errors (see 5.5)
3) Data Reduction Errors (see 5.6)
4) Errors of Method (see 5.7)
5) Subjective or Personal (see 5.8)

The size and complexity of the measurement uncertainty analysis may lead to the use of any or all of these categories.

In most cases, metrological maintenance (calibration, verification, certification) of flowmeters, flow-rate measurements and processing of the data are done by different personnel. To control the possible sources of errors, it is advisable to relate them to the stages of preparation, measurement and processing of the data.

In such cases, it is advisable to classify errors into:

a) calibration errors (see 5.4)

b) errors of measurement or data acquisition errors (see 5.5)

c) errors of processing the measurement data or data reduction errors (see 5.6)

5.4 Calibration errors

The major purpose of the calibration process is to determine systematic errors in order to eliminate them. The calibration process exchanges the large systematic error of an uncalibrated or poorly calibrated instrument for the smaller combination of the systematic error of the standard instrument and the random error of the comparison. This exchange of errors is fundamental and is the basis of the notion that the uncertainty of the standard should be substantially less than that of the test instrument.

Each calibration in the hierarchy constitutes an error source. Figure 8 is a typical transducer calibration hierarchy. Associated with each comparison in the calibration hierarchy is a pair of elemental errors. These errors are the systematic error limit and the sample standard deviation in each process. Note that these elemental errors may be cumulative or independent. For example, B_{21} may include B_{11}. The error sources are listed in table 1. The second digit of the subscript indicates the error category, i.e. 1 indicates calibration error.

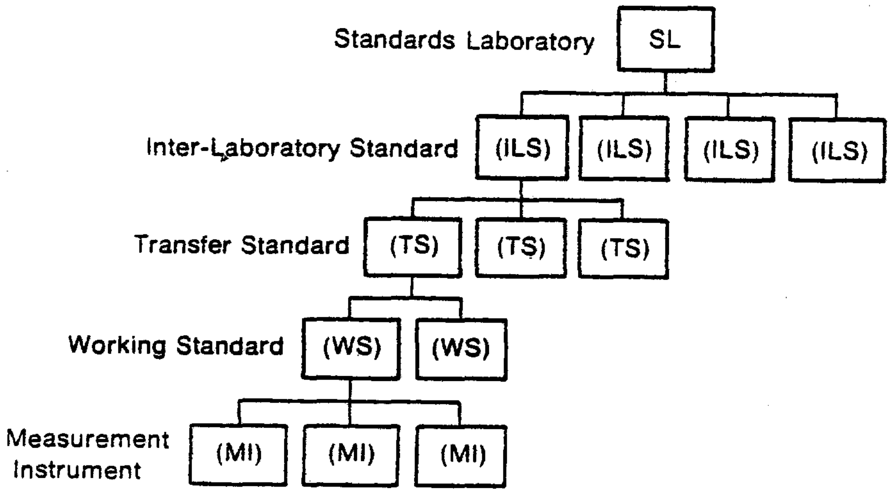


Figure 8 — Basic measurement calibration hierarchy

Table 1 — Calibration hierarchy error sources

Calibration	*Systematic error*	*Experimental standard deviation*	*Degrees of freedom*
SL - ILS	B_{11}	S_{11}	ν_{11}
ILS - TS	B_{21}	S_{21}	ν_{21}
TS - WS	B_{31}	S_{31}	ν_{31}
WS - MI	B_{41}	S_{41}	ν_{41}

5.5 Data acquisition errors

Figure 9 illustrates some of the error sources associated with a typical pressure data acquisition system. Data are acquired by measuring the electrical output resulting from pressure applied to a strain gage type pressure measurement instrument. Other error sources, such as probe errors, including installation effects, and environmental effects, also may be present. The effects of these error sources should be determined by performing overall system calibrations, comparing known applied pressures with measured values. However, should it not be possible to do this, then it is necessary to evaluate each of the elemental errors and combine them to determine the overall error.

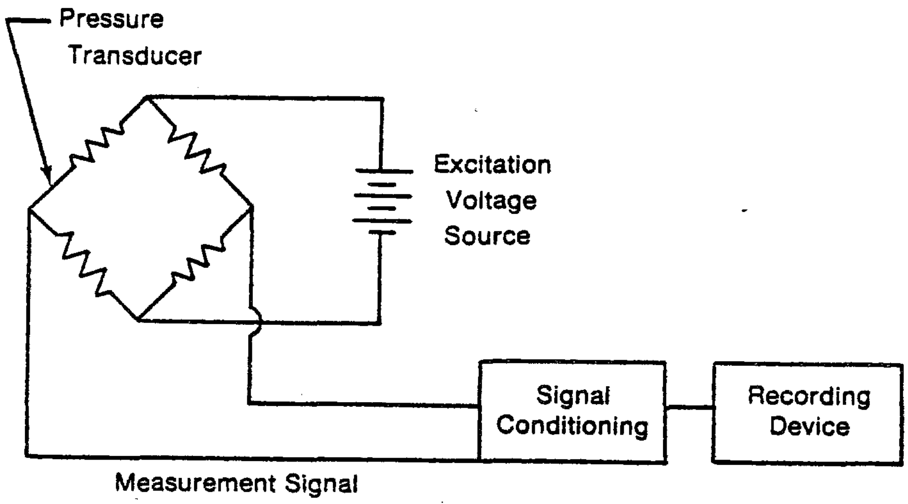


Figure 9 — Data acquisition system

Some of the data acquisition error sources are listed in table 2. Symbols for the elemental systematic and random errors and for the degrees of freedom are shown. Note these elemental errors are independent, not cumulative.

Table 2 — Data acquisition error sources

Error Source	*Systematic error*	*Experimental standard deviation*	*Degrees of freedom*
Excitation Voltage	B_{12}	S_{12}	ν_{12}
Signal Conditioning	B_{22}	S_{22}	ν_{22}
Recording Device	B_{32}	S_{32}	ν_{32}
Pressure Transducer	B_{42}	S_{42}	ν_{42}
Probe Errors	B_{52}	S_{52}	ν_{52}
Environmental Effects	B_{62}	S_{62}	ν_{62}
Spacial Averaging	B_{72}	S_{72}	ν_{72}

5.6 *Data reduction errors*

Computations on raw data produce output in engineering units. Typical errors in this process stem from curve fits and computational resolution. These errors often are negligible.

Symbols for the data reduction error sources are listed in table 3.

Table 3 — Data reduction error sources

Error source	*Systematic error*	*Experimental standard deviation*	*Degrees of freedom*
Curve Fit	B_{13}	S_{13}	ν_{13}
Computational Resolution	B_{23}	S_{23}	ν_{23}

5.7 *Errors of method*

Errors of method are those associated with a particular measurement procedure (principles of use of instruments) and also with the uncertainty of constants used in calculations.

Some examples are errors from indirect methods of flow rate measurement associated with physical inaccuracy of the relationship between the measured quantity and flow-rate, or with inaccuracy of the constants in the relationship. These inaccuracies may be due, for instance, to the fact that the flow conditions prevailing during the measurement are not identical to the conditions in which the calibration has been carried out or for which a standardized discharge coefficient has been established. In certain methods of flow measurement (differential pressure devices for instance), these sources of error arising from the flow conditions are covered by the uncertainty associated with the discharge coefficient, as far as the installation conditions prescribed in the standard are satisfied; if they are not, that Standard does not apply. In other methods (velocity-area method for instance), the uncertainty arising from the flow conditions is identified as a component of the total uncertainty; it shall be evaluated by the user in each case and combined with the other elemental uncertainties.

As a rule, errors of method have a systematic character and can be determined in the course of certification of a flow-rate measuring procedure.

5.8 *Subjective errors*

Subjective errors are caused by personal characteristics of the operators who calibrate flowmeters, perform measurements and process the data. These can include reading errors and miscalculations.

6 Estimation and presentation of elemental errors

6.1 Summary of procedure

Obtain an estimate of each error. If the data is available to estimate the experimental standard deviation, classify the error as a random error. Otherwise, classify it as a systematic error.

Review the test objective, test duration and number of calibrations that will affect the test result. Make the final classification of elemental errors for each measurement. If an error increases the scatter in the measurement result in the defined test, it is a random error; otherwise, it is a systematic error.

6.2 Calculate the experimental standard deviation

There are many ways to calculate the experimental standard deviation:

a) If the parameter to be measured can be held constant, a number of repeated measurements can be used to evaluate equation (1)

$$S = \sqrt{\sum_{i=1}^{N} \frac{(X_i - \overline{X})^2}{N-1}} \qquad (10)$$

b) If there are M redundant instruments or M redundant measurements and the parameter to be measured can be held constant to take N repeat readings, the following pooled estimate of the experimental standard deviation for individual readings can be used:

$$S_{pooled} = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{M}(X_{ji} - \overline{X}_i)^2}{M(N-1)}} \qquad (11)$$

For the experimental standard deviation of the average value of the parameter

$$S_{\bar{x}} = \frac{S_{pooled}}{\sqrt{MN}} \qquad (12)$$

c) If a pair of instruments (providing measurements X_{1i} and X_{2i}) which have the same experimental standard deviation are used to estimate a parameter that is not constant with time, the difference between the readings, Δ, may be used to estimate the experimental standard deviation of the individual instruments as follows:

$$S = \sqrt{\frac{\sum_{i=1}^{N}(\Delta_i - \bar{\Delta})^2}{2(N-1)}} \qquad (13)$$

where

$$\Delta_i = X_{1i} - X_{2i}$$

If the degrees of freedom are less than 30, the small sample methods shown in annex C are required.

6.3 Estimate the systematic error limit

In spite of applying all known corrections to overcome imperfections in calibration, data acquisition and data reduction processes, some systematic errors will probably remain. To determine the exact systematic error in a measurement, it would be necessary to compare the true value and the measurements. However, as the true value is unknown, it is necessary to carry out special tests or utilize existing data that will provide systematic error information. The following examples are given in order of preference.

a) Interlaboratory or interfacility tests make it possible to obtain the distribution of systematic errors between facilities (Reference ISO 5725).

b) Comparisons of standards with instruments in the actual test environment may be used.

c) Comparison of independent measurements that depend on different principles can provide systematic error information. For example, in a gas turbine test, airflow can be measured with (1) an orifice, (2) a bellmouth nozzle, (3) compressor speed-flow rig data, (4) turbine flow parameters and (5) jet nozzle calibrations.

d) When it is known that a systematic error results from a particular cause, calibrations may be performed allowing the cause to perturbate through its complete range to determine the range of systematic error.

e) If there is no source of data for systematic error, the estimate must be based on judgment. An estimate of an upper limit of the systematic error is needed. Instrumentation manufacturers' reports and other references may provide information. It is important to distinguish between the "estimate" of an upper limit on systematic error obtained by this method and the more reliable estimate of a random error arrived at by analyzing data. There is a general tendency to underestimate systematic uncertainties when a subjective approach is used, partly through human optimism and partly through the possibility of overlooking the existence of some sources of systematic error. Great care is therefore necessary when quoting systematic error limits.

Sometimes the physics of the measurement system provide knowledge of the sign but not the magnitude of the systematic error. For example, hot thermocouples radiate and conduct thermal energy away from the sensor to indicate lower temperatures. The systematic error limits in this case are non-symmetrical, i.e., not of the form $\pm B$. They are of the form B^+ for the upper limit and B^- for the lower limit. Thus, typical systematic error limits associated with a radiating thermocouple could be:

$B^+ = 0$ degrees
$B^- = -10$ degrees

For elemental error sources, the interval from B^+ to B^- must include zero.

6.4 Final error classification based on the defined measurement

Uncertainty statements must be related to a well defined measurement process. The final classification of errors into systematic (bias) and random (precision) depends on the definition of the measurement process. Some of these considerations are:

a) Long versus Short Term Testing (see 6.4.1)

b) Comparative versus Absolute Testing (see 6.4.2)

c) Averaging to Reduce Random Error (see 6.4.3)

6.4.1 Long versus short term testing

The calibration histories accumulated before or during the testing period may influence the uncertainty analysis.

1) When the instrumentation is calibrated only once, all the calibration error is frozen into systematic error. The error in the calibration correction is a constant and cannot increase the scatter in a test result. Thus, the calibration error, made up in general of systematic and fossilized random errors, is considered to be all systematic errors in this case.

2) If the test period is long enough that instrumentation may be calibrated several times or more and/or several test stands are involved, the random error in the calibration hierarchy (see 5.4) should be treated as contributing to the overall experimental standard deviation. The experimental standard deviations may be derived from calibration data.

6.4.2 Comparative versus absolute testing

The objective of a comparative test is to determine, with the smallest measurement uncertainty possible, the net effect of a design change. The first test is run with the standard or baseline configuration. The second test is run with the design change. The difference between the results of these tests is an indication of the effect of the design change. As long as only the difference or net effect between the two tests is considered, all systematic errors, being fixed, will cancel out. The measurement uncertainty will be composed of random errors only.

The uncertainty of the back-to-back tests can be considerably reduced by repeating them several times and averaging the differences.

All errors in a comparative test arise from random errors in data acquisition and data reduction. Systematic errors are effectively zero. Since calibration random errors have been considered systematic errors, they also are effectively zero.

The test result is the difference in flow between two test results, r_1 and r_2.

$$\Delta r = r_1 - r_2 \tag{14}$$

and

$$S_{\Delta r} = \sqrt{S_{r_1}^2 + S_{r_2}^2} = \sqrt{2}\, S_{r_1} \tag{15}$$

where S_{r_1} is the random error of the first test, the root sum-square of the experimental standard deviations from data acquisition and data reduction, and S_{r_2} is assumed to equal S_{r_1}.

6.4.3 Averaging to reduce random error

Averaging test results is often used to improve the random uncertainty. Careful consideration should be given to designing the test series to average as many causes of variation as possible within cost constraints. The design should be tailored to the specific situation. For example, if experience indicates time-to-time and rig-to-rig variations are significant, a design that averages multiple test measurement results on one rig on one day may produce optimistic random error estimates compared to testing several rigs, each mounted several times, over a period of weeks. The list of possibilities may include the above plus test stand-to-test stand, instrument-to-instrument, mount-to-mount and environmental, fuel, power and test crew variation. Historic data is invaluable for studying these effects.* If the pretest uncertainty analysis identifies unacceptably large error sources, special tests to measure the effects should be considered.

* A statistical technique, analysis of variance (ANOVA) is useful for partitioning total variance by cause.

6.5 Example: a calibration constant

Assume a test meter is to be compared or calibrated with a master meter at one flow level. The objective is to determine a correction, called a calibration constant, that will be added to the test meter observations when it is installed for test. This calibration constant correction will make the test meter "read like" the master meter. During the calibration, the master meter is used to set the flow level as it is usually more accurate than the test meter. To reduce the calibration random error, N=13 comparisons will be made and averaged. If the data were plotted, the data might look like figure 10.

If the master meter systematic error limit from its own calibration is judged to be no larger than B_M, what will the test meter uncertainty be after calibration?

Define Δ_i = Master Meter Reading$_i$ − Test Meter Reading$_i$

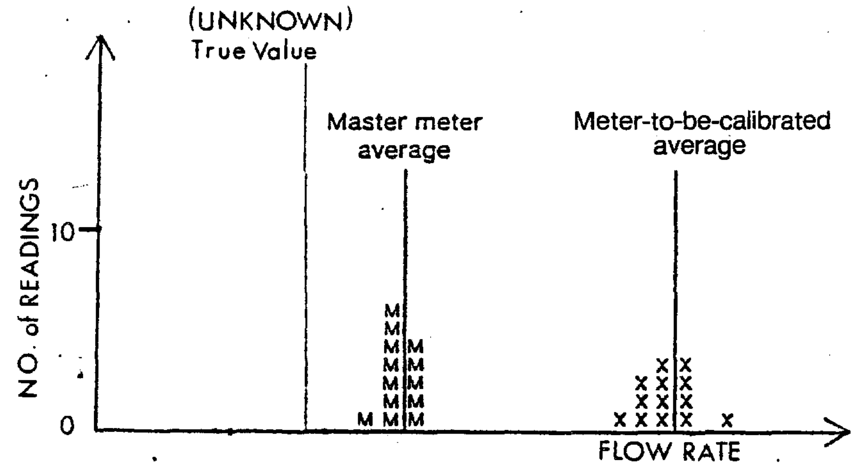
Calibration Constant equals the average

$$K = \bar{\Delta} = \frac{\Sigma \Delta_i}{13} \tag{16}$$

The sample standard deviation of the calibration constant K is:

$$S_K = \frac{S_\Delta}{\sqrt{13}} = \sqrt{\frac{\Sigma(\Delta - \bar{\Delta})^2}{13(12)}} \tag{17}$$

The test meter is later installed in a test stand. Each observation made on the test meter is corrected by adding K. By this process, the error in K from the calibration process is propagated to the corrected data from the test stand.



Master meter systematic error = B_M
Calibration random error = S_K

Figure 10 — Calibration should compensate for test meter systematic error

If the defined measurement process is short, involving a single calibration, K is constant and this error must be a constant or systematic error. It includes the systematic error in the master meter plus the random error in the calibration process. The random error is fossilized into systematic error. The fossilization is indicated by an asterisk. We can estimate an upper limit for this systematic error as:

$$B_K = \sqrt{B_M^2 + (t_{95}\,S_K)^{*2}} \tag{18}$$

Where B_M is the systematic error limit of the master meter and $t_{95} = 2.179$ for 12 degrees of freedom (annex C).)

This calibration systematic error limit would be combined with systematic error limits from data acquisition and data reduction to obtain the measurement systematic error limit. There would also be random error from these last two processes.

If the uncalibrated test meter had a systematic error limit judged to be B_T, the calibration process improved the test accuracy if B_K is less than B_T. Note that the calibration process does not change the test meter random error which is included in the data acquisition random error. However, the test meter random error contributes to the calibration random error S_K. This contribution is reduced by averaging the calibration data.

If the test process is long and involves several calibrations, the calibration error contributes both systematic error (B_M) and random error ($t_{95}\,S_K$) to the final test result.

If the test process is comparative, the difference between two tests with a single calibration, the calibration error is all systematic error and cancels out when one result is subtracted from the other.

7 Combination and propagation errors

7.1 Summary of procedure

Root-sum-square the systematic error limits and experimental standard deviations for each measurement. Propagate the measurement systematic error and random error limits separately all the way to the final test result, either by sensitivity factors or by finitely incrementing the data reduction program. Work consistently in either absolute units or percentages.

7.2 Combining sample standard deviations

The experimental standard deviation (S) of the measurement is the root-sum-square of the elemental experimental standard deviations from all sources, that is;

$$S = \sqrt{\sum_{j=1}^{5}\sum_{i=1}^{k} S_{ij}^2} \tag{19}$$

where j defines the category: such as (1) calibration, (2) data acquisition, (3) data reduction, (4) errors of method and (5) subjective or personal, and i defines the sources within the categories.

For example: the experimental standard deviation for the calibration process in table 1 is:

$$S_1 = S_{Calibration} = \sqrt{\sum_{i=1}^{4} S_{i1}^2} = \sqrt{S_{11}^2 + s_{21}^2 + S_{31}^2 + S_{41}^2} \tag{20}$$

The measurement experimental standard deviation is the root-sum-square of all the elemental experimental standard deviations in the measurement system:

$$S = S_{Measurement} = \sqrt{\sum_{j=1}^{3} S_j^2} = \sqrt{S_1^2 + S_2^2 + S_3^2} \tag{21}$$

Categories (4) or (5) are optional and may or may not be employed.

7.3 Combining elemental systematic error limits

If there were only a few sources of elemental systematic errors, it might be reasonable to add them together to obtain the overall systematic error limits. For example, if there were three sources, the probability that they would all be plus (or minus) would be one-half raised to the third power or one eighth. However, the probability that all three will have the same sign and be at the systematic error limit is extremely small. In actual practice, most measurements will have ten, twenty or more sources of systematic error. The probability that they would all be plus (or minus) and be at their limit is close to zero, and therefore, it is more appropriate to combine them by root-sum-square.

If a measurement uncertainty analysis identifies four or less sources of systematic error, there should be some concern that some sources have been overlooked. The analysis should be redone and expert help should be recruited to examine the calibration hierarchy, the data acquisition process and the data reduction procedure for additional sources.

Therefore, the systematic error limit will be used herein as the root-sum-square of the elemental errors from all sources.

$$B = \sqrt{\sum_j \sum_i B_{ij}^2} \tag{22}$$

For example: the systematic error limit for the calibration hierarchy (table 1) is

$$B_1 = B_{Cal} = \sqrt{B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2} \tag{23}$$

The systematic error limit for the basic measurement process is

$$B = \sqrt{B_1^2 + B_2^2 + B_3^2} \tag{24}$$

If any of the elemental systematic error limits are non-symmetrical, separate root-sum-squares are used to obtain B^+ and B^-. For example, assume B_{21} and B_{23} are non-symmetrical, i.e. B_{21}^+, B_{21}^-, B_{23}^+ and B_{23}^- are available. Then

$$B^+ = \sqrt{B_{11}^2 + (B_{21}^+)^2 + B_{31}^2 + B_{41}^2 + B_2^2 + B_{13}^2 + (B_{23}^+)^2} \tag{25}$$

$$B^- = \sqrt{B_{11}^2 + (B_{21}^-)^2 + B_{31}^2 + B_{41}^2 + B_2^2 + B_{13}^2 + (B_{23}^-)^2} \tag{26}$$

7.4 Propagation of measurement errors

Fluid flow parameters are rarely measured directly; usually more basic quantities such as temperature and pressure are measured, and the fluid flow parameter is calculated as a function of the measurements. Error in the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with the Taylor's series methods. It is convenient to introduce the concept of the sensitivity of a result to a measured quantity as the error propagated to the result due to unit error in the measurement. The "sensitivity coefficient" (also known as "influence coefficient") of each subsidiary quantity is most easily obtained in one of two ways.

a) Analytically

Where there is a known mathematical relationship between the result, R, and subsidiary quantities, Y_1, Y_2 . . . Y_K the dimensional sensitivity coefficient, θ_i of the quantity Y_i, is obtained by partial differentiation. Thus, if $R = f(Y_1, Y_2 \ldots Y_K)$, then

$$\theta_j = \frac{\partial R}{\partial Y_i} \tag{27}$$

Analogously, the relative (nondimensional) sensitivity coefficient, θ_i', is

$$\theta_i' = \frac{\partial R/R}{\partial Y_i/Y_i} \tag{28}$$

In this form, the sensitivity is expressed as "percent/percent." That is, θ_i' is the percentage change in R brought about by a 1% change in Y_i. This is the form to be used if the uncertainties to be combined are expressed as percentages of their associated variables rather than absolute values.

b) Numerically

Where no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate θ_i. This is a convenient method with computer calculations.

Here θ_i is given by

$$\theta_i = \frac{\Delta R}{\Delta Y_i} \tag{29}$$

The result is calculated using Y_i to obtain R, and then recalculated using $(Y_i + \Delta Y)$ to obtain $(R + \Delta R)$. The value of ΔY used should be as small as practicable.

Care should be taken to ensure that the errors are independent. With complex parameters, the same measurement may be used more than once in the formula. This may increase or decrease the error depending on whether the sign of the measurement is the same or opposite. If the Taylor's series relates the most elementary mesurements to the ultimate parameter or result, these "linked" relationships will be properly accounted for.

This effect can be covered by calculating a modified θ by simultaneous perturbation of all the inputs likely to be affected, thus:

θ_{link} = (Change in output R due to simultaneous application of linked error in all inputs, y_i)

An example of this is barometric pressure which affects all pressure inputs simultaneously, in a "gauge-pressure" system. Another example is the use of a common working standard to calibrate all the pressure transducers.

Such linked errors can then be combined with independent ones, thus:

$$S(R) = \sqrt{[\theta_{link}\, S(y_{link})]^2 + \Sigma\, [\theta_i\, S(y_i)\,]^2} \tag{30}$$

7.5 Airflow example

In this example, airflow is determined by the use of a sonic nozzle and measurements of upstream stagnation temperature and stagnation pressure (figure 11).

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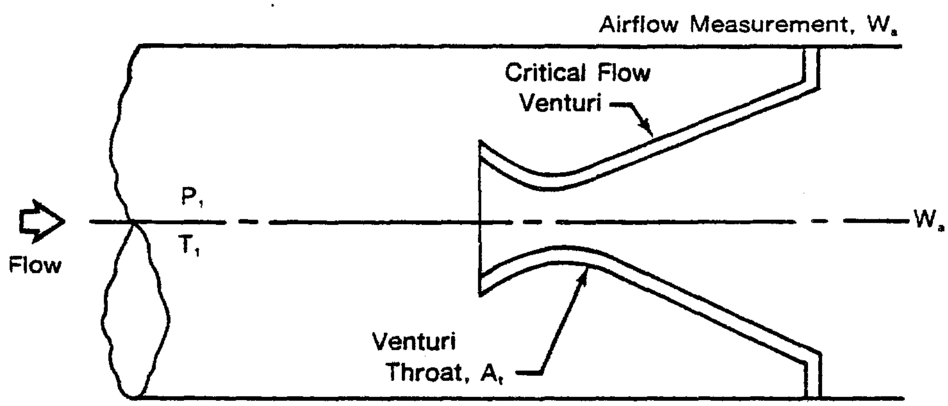

Figure 11 — Flow through a sonic nozzle

The flow is calculated from

$$W = CaF_a\varphi^* \frac{P_{1t}}{\sqrt{T_{1t}}} \tag{31}$$

where

$\dot{W}$ is the mass flowrate of air
F_a is the factor to account for thermal expansion of the venturi
a is the venturi throat area
P_{1t} is the total (stagnation) pressure upstream
T_{1t} is the total temperature upstream
φ^* is the factor to account for the properties of the air (critical flow constant)
C is the discharge coefficient

The experimental standard deviation for the Flow (S_w) is calculated using the Taylor's series expansion.

Assuming C equals 1 and has negligible error

$$S_w = \left[(\theta_{F_a} S_{F_a})^2 + (\theta_{\varphi^*} S_{\varphi^*})^2 + (\theta_a S_a)^2 + (\theta_{P_{1t}} S_{P_{1t}})^2 + (\theta_{T_{1t}} S_{T_{1t}})^2 \right]^{1/2} \tag{32}$$

where

$$\frac{\partial W}{\partial F_a}$$

denotes the partial derivative of W with respect to F_a.

$$S_w = C\left[\left(\frac{\varphi^* a P_{1t}}{\sqrt{T_{1t}}} S_{F_a}\right)^2 + \left(\frac{F_a a P_{1t}}{\sqrt{T_{1t}}} S_{\varphi^*}\right)^2 + \left(\frac{F_a \varphi^* P_{1t}}{\sqrt{T_{1t}}} S_a\right)^2 + \left(\frac{F_a \varphi^* a}{\sqrt{T_{1t}}} S_{P_{1t}}\right)^2 + \left(\frac{F_a \varphi^* a P_{1t}}{-2\sqrt{T_{1t}^3}} S_{T_{1t}}\right)^2 \right]^{1/2} \tag{33}$$

By inserting the values and random errors from table 4 into equation (32), the random error of 0.17 kg/sec for airflow is obtained.

The systematic error in the flow calculation is propagated from the systematic error limits of the measured variables. Using the Taylor's series formula gives

$$B_w = \left[(\theta_{x_1} B_{x_1})^2 + (\theta_{x_2} B_{x_2})^2 + (\theta_{x_3} B_{x_3})^2 + \ldots + (\theta_{x_m} B_{x_m})^2 \right]^{1/2} \tag{34}$$

For this example,

$$B_w = \left[(\theta_{F_a} B_{F_a})^2 + (\theta_{\varphi^*} B_{\varphi^*})^2 + (\theta_a B_a)^2 + (\theta_{P_{1t}} B_{P_{1t}})^2 + (\theta_{T_{1t}} B_{T_{1t}})^2 \right]^{1/2} \tag{35}$$

Taking the necessary partial derivatives gives

$$
\begin{aligned}
B_w = C\Bigg[& \left(\frac{\varphi^* a P_{1t}}{\sqrt{T_{1t}}} B_{F_a}\right)^2 + \left(\frac{F_a a P_{1t}}{\sqrt{T_{1t}}} B_{\varphi^*}\right)^2 \\
& + \left(\frac{F_a \varphi^* P_{1t}}{\sqrt{T_{1t}}} B_a\right)^2 + \left(\frac{F_a \varphi^* a}{\sqrt{T_{1t}}} B_{P_{1t}}\right)^2 \\
& + \left(\frac{F_a \varphi^* a P_{1t}}{-2\sqrt{T_{1t}^3}} B_{T_{1t}}\right)^2 \Bigg]^{1/2}
\end{aligned} \quad (36)
$$

By inserting the values and systematic error limits of the measured parameters from table 4 into equation (36), a systematic error limit of 0.32 kg/sec is obtained for a nominal airflow of 112.64 kg/sec.

Table 4 contains a summary of the measurement uncertainty analysis for this flow measurement. It should be noted the errors listed only apply to the nominal values.

Table 4 — Flow data

Parameter	*Units*	*Nominal value*	*Experimental standard deviation (one experimental standard deviation)*	*Systematic error*
F_a	—	1.00	0.0	0.001
C	—	1.0	0.0	0.0
φ^*	kg $K^{1/2}$ newton sec	0.0404	0.0	4.04×10^{-3}
a	m^2	0.191	9.55×10^{-5}	3.82×10^{-4}
P_{1t}	Pa	2.54×10^5	345.0	345.0
T_{1t}	K	303.0	0.17	0.17
∴ w	kg/sec	112.64	0.17	0.32

8 Calculation of uncertainty

8.1 Summary of procedure

Select U_{ADD} and/or U_{RSS} and combine the systematic and random errors of the test result to obtain the uncertainty. The test result plus and minus the uncertainty is the uncertainty interval that should contain the true value with high probability.

8.2 Uncertainty intervals

For simplicity of presentation, a single number (some combination of systematic and random errors) is needed to express a reasonable limit for error. The single number should have a simple interpretation (like the largest error reasonably expected) and be useful without complex explanation. It is usually impossible to define a single rigorous statistic because the systematic error is an upper limit based on judgment which has unknown characteristics.* This function is a hybrid combination of an unknown quantity (systematic error) and a statistic (random error). If both numbers were statistics, a confidence interval would be recommended. 95% or 99% confidence levels would be available at the discretion of the analyst. Although rigorous statistical confidence levels are not available, two uncertainty intervals, approximately analogous to 95% and 99% levels, are recommended. This analogy is discussed in Annex F.

8.3 Symmetrical intervals

Uncertainty (figure 12) for the symmetrical systematic error case is centered about the measurement and the uncertainty intervals are defined as:

$R - U, R + U$, where

$$U_{ADD} = U_{99} = (B + t_{95} S) \quad (37)$$

$$U_{RSS} = U_{95} = \sqrt{B^2 + (t_{95} S)^2} \quad (38)$$

If the sample standard deviation is based on small samples, the methods in annex C may be used to determine a value of Student's t_{95}. For large samples (>30), 2 may be substituted for t_{95} in equations (37) and (38).

If the test result is an average $(\overline{R})$ based on sample size N, instead of a single value (R), $S/\sqrt{N}$ should be substituted for S.

The uncertainty interval selected (equations (37) or (38)) should be provided in the presentation; the components (systematic error, random error, degrees of freedom) should be available in an appendix or in supporting documentation. These three components may be required to substantiate and explain the uncertainty value, to provide a sound technical base

* If information exists to justify the assumption that the systematic error limits have a random distribution, a rigorous statistic can be defined as shown in annex E.

for improved measurements, and to propagate the uncertainty from measured parameters to fluid flow parameters and from fluid flow parameters to other more complex performance parameters (i.e., fuel flow to Thrust Specific Fuel Consumption (TSFC), TSFC to aircraft range, etc).

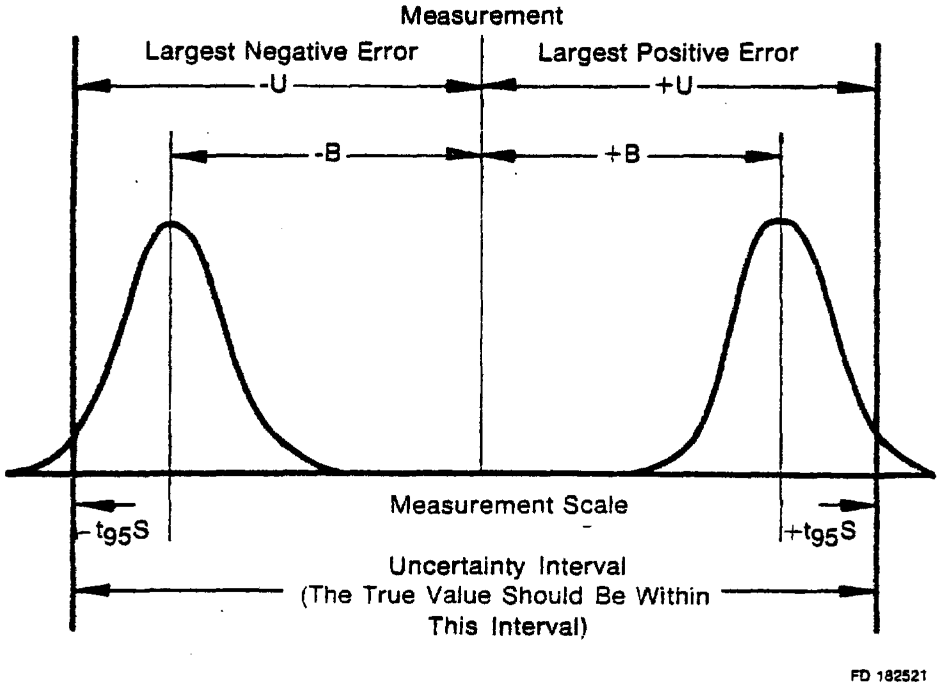

Figure 12 — Measurement uncertainty interval (U_{99}); symmetrical systematic error

9 Presentation of results

9.1 *Summary of requirement*

The summary report should contain the nominal level of the test result, the systematic error, the sample standard deviation, the degrees of freedom and the uncertainty. The equation used to calculate uncertainty, U_{ADD} or U_{RSS} should be stated. The summary should reference a table of the elemental errors considered and included in the uncertainty.

9.2 *Reporting error summary*

The definition of the components, systematic error limit, experimental standard deviation and the limit (U) suggests a summary format for reporting measurement error. The format will describe the components of error, which are necessary to estimate further propagation of the errors, and a single value (U) which is the largest error expected from the combined errors. Additional information, degrees of freedom for the estimate of S, is required to use the experimental standard deviation if small samples were used to calculate S. These summary numbers provide the information necessary to accept or reject the measurement error. The reporting format is:

a) S, the estimate of the experimental standard deviation, calculated from data.

b) For small samples, ν, the degrees of freedom associated with the estimate of the experimental standard deviation (S). The degrees of freedom for small samples (less than 30) is obtained from the Welch-Satterthwaite procedure illustrated in annex C.

c) B, the upper limit of the systematic error of the measurement process or B^- and B^+ if the systematic error limit is non-symmetrical.

d) The uncertainty formula should be stated. $U_{99} = (B + t_{95}\,S)$ or $U_{95} = \sqrt{B^2 + (t_{95}\,S)^2}$, the uncertainty interval, within which the error should fall. If the systematic error limit is non-symmetrical, $U_{99}^{-} = B^{-} - t_{95}S$ and $U_{99}^{+} = B^{+} + t_{95}\,S$. No more than two significant places should be reported. For small samples see annex C.

The model components, S, ν, B, and U, are required to report the error of any measurement process. The first three components, S, ν, and B, are necessary to: (1) indicate corrective action if the uncertainty is unacceptably large before the test, (2) to propagate the uncertainty to more complex parameters, and (3) to substantiate the uncertainty limit.

9.3 Reporting error — table of elemental sources

To support the measurement uncertainty summary, a table detailing the elemental error sources is needed for several purposes. If corrective action is needed to reduce the uncertainty or to identify data validity problems, the elemental contributions are required. Further, if the uncertainty quoted in the summary appears to be optimistically small, the list of sources considered should be reviewed to identify missing sources. For this reason, it is important to list all sources considered even if negligible.

Note that all errors in table 5 have been propagated from the basic measurement to the end result before listing and, therefore, they are expressed in units of the test result.

Table 5 — Elemental error sources

	ij subscript	*Source*	*Measurement nominal value*	*Experimental standard deviation S_{ij}*	*Degrees of freedom ν_{ij}*	*Systematic error limit B_{ij}*	*Source of systematic error*	
Calibration	11							
	21							
	31							
	—							
	—							
	—							
Data Acquisition	12							
	22							
	32							
	42							
	—							
	—							
	—							
Data Reduction	13							
	23							
	33							
	—							
	—							
	—							
	Results		Nominal Value ____	$S = \sqrt{\Sigma S_{ij}^{2}}$ ____	ν w/s ____	$B = \sqrt{\Sigma B^2}$ ____		$t_{95\%}$ ____

$$U_{99} = B + t_{95}\,(S) = \_\_\_$$

$$U_{95} = \sqrt{B^2 + (t_{95}S)^2} = \_\_\_$$

9.4 Pre-test analysis and corrective action

Uncertainty is a function of the measurement process. It provides an estimate of the largest error that may reasonably be expected for that measurement process. Errors larger than the uncertainty should rarely occur. If the difference to be detected in an experiment is of the same size or smaller than the projected uncertainty, corrective action should be taken to reduce the uncertainty. Therefore, it is recommended that an uncertainty analysis always be done before the test or experiment. The recommended corrective action depends on whether the systematic or the random error is too large as shown in table 6.

Table 6 — Recommended corrective action if the predicted pretest measurement accuracy is unacceptable

Systematic Error Limit Too Large:	*Random Error Too Large:*
• Improve calibration	• Larger test sample
• Independent calibrations for redundant meters	• More precise instrumentation
• Concommitant variable	• Redundant instrumentation
• In place calibration	• Data smoothing — Moving average — Filter — Regression
	• Improve design of experiment

9.5 Post-test analysis and data validity

Post-test analysis is required to confirm the pretest estimates or to identify data validity problems. Comparison of measurement test results with the pretest analysis is an excellent data validity check. The random error of the repeated points or redundant instruments should not be significantly larger than the pretest measurement estimates. When redundant instrumentation or calculation methods are available, the individual uncertainty intervals should be compared for consistency with each other and with the pretest measurement uncertainty analysis.

Three cases are illustrated in figure 13.

When there is no overlap between uncertainty intervals, as in Case I, a problem exists. The true value cannot be contained within both intervals. That is, there should be a very low probability that the true value lies outside any of the uncertainty intervals. Either the uncertainty analysis is wrong or a data validity problem exists. Investigation to identify bad readings, overlooked systematic error, etc., is necessary to resolve this discrepancy. Redundant and dissimilar instrumentation should be compared. Partial overlap of the uncertainty intervals, as in Case II, also signals that a problem may exist. The magnitude of the problem depends on the amount of overlap. The only situation when one can be confident that the data is valid and the uncertainty analysis is correct is Case III, when the uncertainty intervals completely overlap.

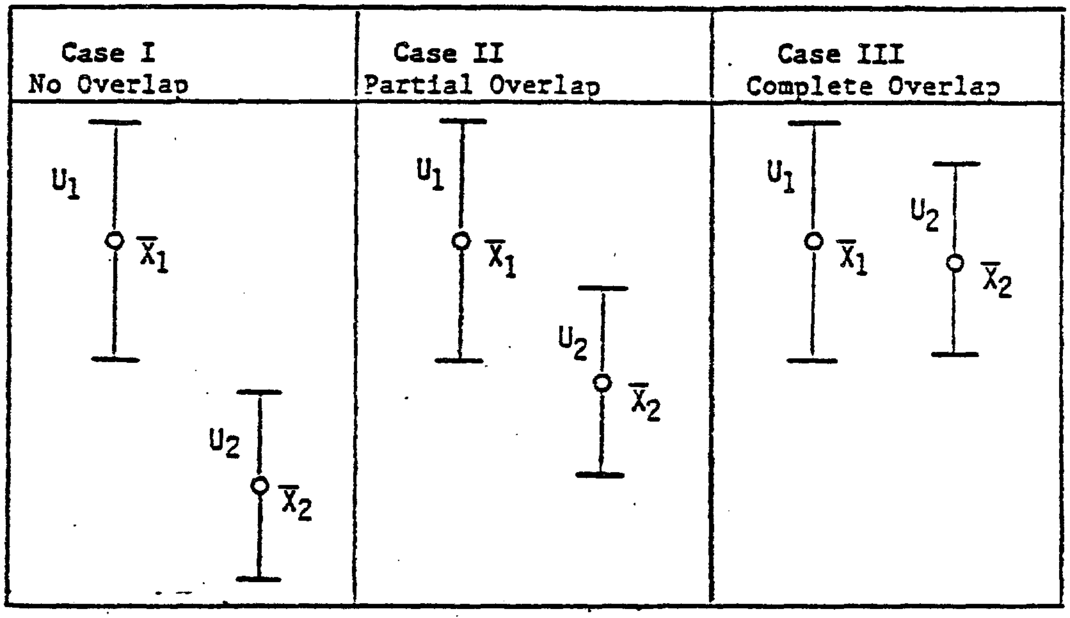

Figure 13 — Three post-test measurement uncertainty interval comparisons

Annex A: Examples on estimation of uncertainty in airflow measurement

Introduction

This annex contains three examples of fluid flow measurement uncertainty analysis. The first deals with airflow measurement for an entire facility (with several test stands) over a long period. It also applies to a single test with a single set of instruments. The second example demonstrates how comparative development tests can reduce the uncertainty of the first example. The third example illustrates a liquid flow measurement.

A.1 General

Airflow measurements in gas turbine engine systems are generally made with one of three types of flowmeters: venturis, nozzles and orifices. Selection of the specific type of flowmeter to use for a given application is contingent upon a tradeoff between measurement accuracy requirements, allowable pressure drop and fabrication complexity and cost.

Flowmeters may be further classified into two categories: subsonic flow and critical flow. With a critical flowmeter, in which sonic velocity is maintained at the flowmeter throat, mass flowrate is a function only of the upstream gas properties. With a subsonic flowmeter, where the throat Mach number is less than sonic, mass flowrate is a function of both upstream and downstream gas properties.

Equations for the ideal mass flowrate through nozzles, venturies and orifices are derived from the continuity equation:

$$W = \rho a V \tag{39}$$

In using the continuity equation as a basis for ideal flow equation derivations, it is normal practice to assume conservation of mass and energy and one-dimensional isentropic flow. Expressions for ideal flow will not yield actual flow since actual conditions always deviate from ideal. An empirically determined correction factor, the discharge coefficient (C) is used to adjust ideal to actual flow:

$$C = W_{actual}/W_{ideal} \tag{40}$$

A.2 Example one — test facility

A.2.1 Definition of the measurement process

What is the airflow measurement capability of a given industrial or government test facility? This question might relate to a guarantee in a product specification or a research contract. Note that this question implies that many test stands, sets of instrumentation and calibrations over a long period of time should be considered.

The same general uncertainty model is applied in the second example to a single stand process, the comparative test.

These examples will provide, step by step, the entire process of calculating the uncertainty of the airflow parameter. The first step is to understand the defined measurement process and then identify the source of every possible error. For each measurement, calibration errors will be discussed first, then data acquisition errors, data reduction errors, and finally, propagation of these errors to the calculated parameter.

Figure 14 depicts a critical venturi flowmeter installed in the inlet ducting upstream of a turbine engine under test for this example.

When a venturi flowmeter is operated at critical pressure ratios, i.e., (P_2/P_1) is a minimum, the flowrate through the venturi is a function of the upstream conditions only and may be calculated from

$$W = \frac{\pi d^2}{4} C F_a \varphi^* \frac{P_1}{\sqrt{T_1}} \tag{41}$$

A.2.2 Measurement error sources

Each of the variables in equation 41 must be carefully considered to determine how and to what extent errors in the determination of the variable affect the calculated parameter. A relatively large error in some will affect the final answer very little, whereas small errors in others have a large effect. Particular care should be taken to identify measurements that influence the fluid flow parameters in more than one way.

In equation (41), upstream pressure and temperature (P_1 and T_1) are of primary concern. Error sources for each of these measurements are: (1) calibration, (2) data acquisition and (3) data reduction.

A.2.2.1 Figure 15 illustrates a typical calibration hierarchy. Associated with each comparison in the calibration hierarchy is a possible pair of elemental errors, a systematic error limit and an experimental standard deviation. Table 7 lists all of the elemental errors. Note that these elemental errors are not cumulative, e.g., B_{21} is not a function of B_{11}. The systematic error limits should be based on interlaboratory tests if available, otherwise, the judgment of the best experts must be used. The experimental standard deviations are calculated from calibration history data banks.

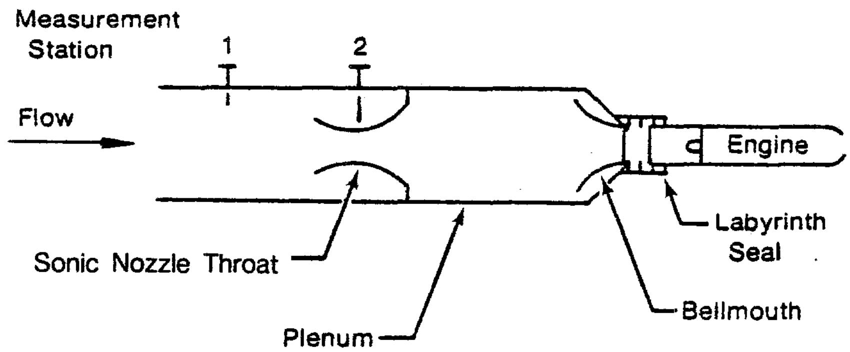

Figure 14 — Schematic of sonic nozzle flowmeter installation upstream of a turbine engine

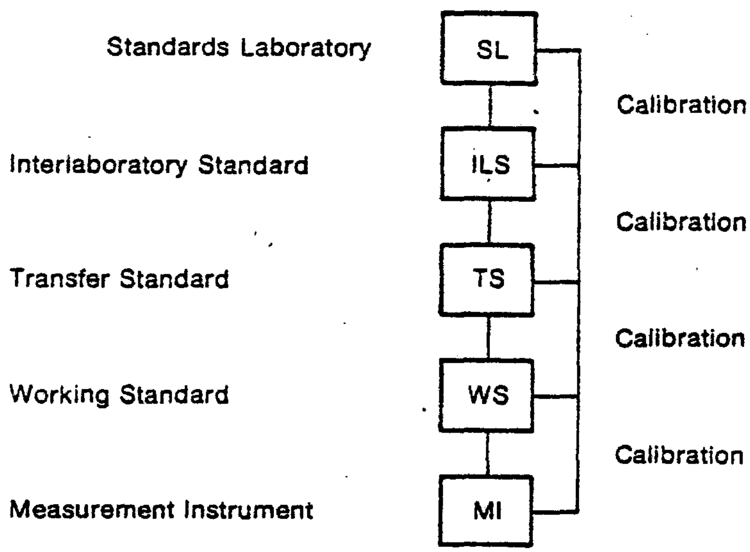

Figure 15 — Typical calibration hierarchy

Table 7 — Calibration hierarchy error sources

Calibration	*Systematic error, P_a*	*Experimental standard deviation, P_a*	*Degrees of freedom*
SL - ILS	B_{11} = 68.953	S_{11} = 13.787	ν_{11} = 10
ILS-TS	B_{21} = 68.953	S_{21} = 13.787	ν_{21} = 15
TS - WS	B_{31} = 68.953	S_{31} = 13.787	ν_{31} = 20
WS - MI	B_{41} = 124.	S_{41} = 36.541	ν_{41} = 30

The experimental standard deviation for the calibration process is the root-sum-square of the elemental sample standard deviations, i.e.,

$$S_1 = \sqrt{S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2}$$

$$= \sqrt{13.787^2 + 13.787^2 + 13.787^2 + 36.541^2}$$

$$= 43.65 \text{ P}_a \tag{42}$$

Degrees of freedom associated with S are calculated from the Welch-Satterthwaite formula as follows:

$$\nu_1 = \frac{(S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2)^2}{\left(\frac{S_{11}^4}{\nu_{11}} + \frac{S_{21}^4}{\nu_{21}} + \frac{S_{31}^4}{\nu_{31}} + \frac{S_{41}^4}{\nu_{41}}\right)}$$

$$= \frac{(13.787^2 + 13.787^2 + 13.787^2 + 36.541^2)^2}{\left(\frac{13.787^4}{10} + \frac{13.787^4}{15} + \frac{13.787^4}{20} + \frac{36.541^4}{30}\right)} = 54. \tag{43}$$

The systematic error for the calibration process is the root-sum-square of the elemental systematic error limits, i.e.,

$$B_1 = \sqrt{B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2} \tag{44}$$

$$= \sqrt{68.953^2 + 68.953^2 + 68.953^2 + 124.117^2}$$

$$= 172.2 \text{ P}_a \tag{45}$$

Data acquisition error sources for pressure measurement are listed in table 8.

Table 8 — Pressure transducer data acquisition error sources

Error source	*Systematic error, P_a*	*Experimental standard Deviation, P_a*	*Degrees of freedom*
Excitation Voltage	B_{12} = 68.953	S_{12} = 34.481	ν_{12} = 40
Electrical Simulation	B_{22} = 68.953	S_{22} = 34.481	ν_{22} = 90
Signal Conditioning	B_{32} = 68.953	S_{32} = 34.481	ν_{32} = 200
Recording Device	B_{42} = 68.953	S_{42} = 34.481	ν_{42} = 10
Pressure Transducer	B_{52} = 68.953	S_{52} = 48.270	ν_{52} = 100
Environmental Effects	B_{62} = 68.953	S_{62} = 68.953	ν_{62} = 10
Probe Errors	B_{72} = 117.223	S_{72} = 48.270	ν_{72} = 60

The experimental standard deviation for the data acquisition process is

$$S_2 = \sqrt{S_{12}^2 + S_{22}^2 + S_{32}^2 + S_{42}^2 + S_{52}^2 + S_{62}^2 + S_{72}^2}$$

$$S_2 = [\, 34.481^2 + 34.481^2 + 34.481^2 + 34.481^2 + 48.270^2 + 68.953^2 + 48.270^2 \,]^{1/2}$$

$$= 119.039 \text{ P}_a \tag{46}$$

$$\nu_2 = \frac{(S_{12}^2 + S_{22}^2 + S_{32}^2 + S_{42}^2 + S_{52}^2 + S_{62}^2 + S_{72}^2)^2}{\left(\frac{S_{12}^4}{\nu_{12}} + \frac{S_{22}^4}{\nu_{22}} + \frac{S_{32}^4}{\nu_{32}} + \frac{S_{42}^4}{\nu_{42}} + \frac{S_{52}^4}{\nu_{52}} + \frac{S_{62}^4}{\nu_{62}} + \frac{S_{72}^4}{\nu_{72}}\right)}$$

$$\nu_2 = (34.481^2 + 34.481^2 + 34.481^2 + 34.481^2 + 48.270^2 + 68.953^2 + 48.270^2)^2 \Big/ \left(\frac{34.481^4}{40} + \frac{34.481^4}{90} + \frac{34.481^4}{200} + \frac{34.481^4}{10} + \frac{48.270^4}{100} + \frac{68.953^4}{10} + \frac{48.270^4}{60}\right) = 77 \tag{47}$$

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The systematic error limit for the data acquisition process is x

$$B_2 = \left[68.953^2 + 68.953^2 + 68.953^2 + 68.953^2 \right]^{1/2}$$

$$+ 68.953^2 + 68.953^2 + 117.223^2$$

$$= 205.6 \text{ P}_a \tag{48}$$

A computer operates on raw pressure measurement data to perform the conversion to engineering units. Errors in this process are called data reduction errors and stem from curve fits and computer resolution.

Computer resolution is the source of a small elemental error. Some of the smallest computers used in experimental test applications have six digits resolution. The resolution error is then plus or minus one in 10^6. Even though this error is probably negligible, consideration should be given to rounding off and truncating errors. Rounding-off results in a random error. Truncating always results in a systematic error (assumed in this example.)

Table 9 lists data reduction error sources.

Table 9 — Pressure measurement data reduction error sources

Error source	*Systematic error, P_a*	*Experimental standard deviation, P_a*	*Degrees of freedom*
Curve Fit	$B_{13} = 68.953$	$S_{13} = 0$	v_{13}
Computer Resolution	$B_{23} = 6.894$	$S_{23} = 0$	v_{23}

The experimental standard deviation for the data reduction process is

$$S_3 = \sqrt{S_{13}^2 + S_{23}^2}$$

$$= 0.0 \tag{49}$$

The systematic error limit for the data reduction process is

$$B_3 = \sqrt{B_{13}^2 + B_{23}^2}$$

$$B_3 = \sqrt{68.953^2 + 6.894^2}$$

$$= 69.297 \text{ P}_a \tag{50}$$

The experimental sample standard deviation for pressure measurement then is

$$S_p = \left[S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2 + S_{12}^2 + S_{22}^2 + S_{32}^2 \right.$$

$$\left. + S_{42}^2 + S_{52}^2 + S_{62}^2 + S_{72}^2 + S_{13}^2 + S_{23}^2 \right]^{1/2} \tag{51}$$

or

$$S_p = \sqrt{S_1^2 + S_2^2 + S_3^2}$$

$$= \sqrt{43.651\,9^2 + 119.039^2 + 0.0^2}$$

$$= 126.790 \text{ P}_a \tag{52}$$

Degrees of freedom associated with the experimental standard deviation are determined as follows:

$$v_p = (S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2 + S_{12}^2 + S_{22}^2 + S_{32}^2 + S_{42}^2 + S_{52}^2 + S_{62}^2$$

$$+ S_{72}^2 + S_{13}^2 + S_{23}^2)^2$$

$$\Big/ \left(\frac{S_{11}^4}{v_{11}} + \frac{S_{21}^4}{v_{21}} + \frac{S_{31}^4}{v_{31}} + \frac{S_{41}^4}{v_{41}} + \frac{S_{12}^4}{v_{12}} + \frac{S_{22}^4}{v_{22}} + \frac{S_{32}^4}{v_{32}} + \frac{S_{42}^4}{v_{42}} \right.$$

$$\left. + \frac{S_{52}^4}{v_{52}} + \frac{S_{62}^4}{v_{62}} + \frac{S_{72}^4}{v_{72}} + \frac{S_{13}^4}{v_{13}} + \frac{S_{23}^4}{v_{23}} \right) \tag{53}$$

or

$$\nu_p = \frac{(S_1^2 + S_2^2 + S_3^2)^2}{\left(\frac{S_1^4}{\nu_1} + \frac{S_2^4}{\nu_2} + \frac{S_3^4}{\nu_3}\right)}$$

$$\nu_p = \frac{(43.651\,9^2 + 119.039^2 + 0.0^2)^2}{\left(\frac{43.651\,9^2}{54} + \frac{119.039^2}{77} + \frac{0.0^4}{0}\right)}$$

$$= 96 \text{ therefore } t_{95} = 2. \tag{54}$$

The systematic error limit for the pressure measurement is

$$B_p = [\, B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2 + B_{12}^2 + B_{22}^2 + B_{32}^2 + B_{42}^2 + B_{52}^2 + B_{62}^2 + B_{72}^2 + B_{13}^2 + B_{23}^2]^{1/2} \tag{55}$$

or

$$B_p = \sqrt{B_1^2 + B_2^2 + B_3^2}$$

$$B_p = \sqrt{172.246^2 + 205.593^2 + 69.297^2}$$

$$= 277.018 \text{ P}_a \tag{56}$$

Uncertainty for the pressure measurement is

$$U_{99} = (B_p + t_{95} S_p),\; U_{95} = \sqrt{B_p^2 + (t_{95} S_p)^2}$$

$$U_{99} = (277.018 + 2 \times 126.790)$$

$$= 530.598 \text{ P}_a,\; U_{95} = 375.6 \text{ P}_a \tag{57}$$

A.2.2.2 The calibration hierarchy for temperature measurements is similar to that for pressure measurements. Figure 16 depicts a typical temperature measurement hierarchy. As in the pressure calibration hierarchy, each comparison in the temperature calibration hierarchy may produce elemental systematic and random errors. Table 10 lists temperature calibration hierarchy elemental errors.

Table 10 — Temperature calibration hierarchy elemental errors

Calibration	*Systematic error, K*	*Experimental standard deviation, K*	*Degrees of freedom*
SL - ILS	$B_{11} = 0.056$	$S_{11} = 0.002$	$\nu_{11} = 2$
ILS-TS	$B_{21} = 0.278$	$S_{21} = 0.028$	$\nu_{21} = 10$
TS - WS	$B_{31} = 0.333$	$S_{31} = 0.028$	$\nu_{31} = 15$
WS - MI	$B_{41} = 0.378$	$S_{41} = 0.039$	$\nu_{41} = 30$

The calibration hierarchy experimental standard deviation is calculated as

$$S_1 = \sqrt{S_{12}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2}$$

$$= \sqrt{0.002^2 + 0.028^2 + 0.028^2 + 0.039^2}$$

$$= 0.056 \text{ °K}. \tag{58}$$

Degrees of freedom associated with S_1 are

$$\nu_1 = \frac{(S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2)^2}{\left(\frac{S_{11}^4}{\nu_{11}} + \frac{S_{21}^4}{\nu_{21}} + \frac{S_{31}^4}{\nu_{31}} + \frac{S_{41}^4}{\nu_{41}}\right)}$$

$$= \frac{(0.002^2 + 0.028^2 + 0.028^2 + 0.039^2)^2}{\left(\frac{0.002^4}{2} + \frac{0.028^4}{10} + \frac{0.028^4}{15} + \frac{0.039^4}{30}\right)}$$

$$= 53 > 30, \text{ therefore } t_{95} = 2. \tag{59}$$

The calibration hierarchy systematic error limit is

$$B_1 = \sqrt{B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2} \tag{60}$$

$$= \sqrt{0.056^2 + 0.278^2 + 0.333^2 + 0.378^2} \tag{61}$$

$$= 0.578 \text{ °K}.$$

A reference temperature monitoring system will provide an excellent source of data for evaluating both data acquisition and reduction temperature random errors.

Figure 17 depicts a typical setup for measuring temperature with Chromel-Alumel thermocouples.

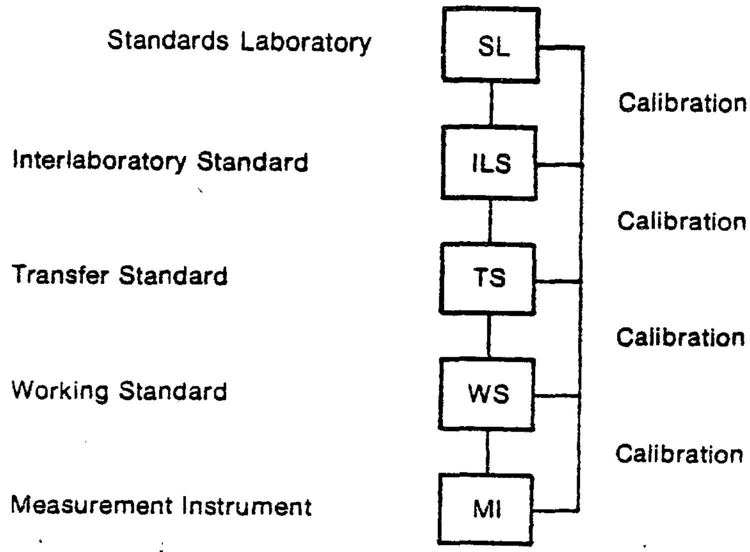

Figure 16 — Temperature measurement calibration hierarchy

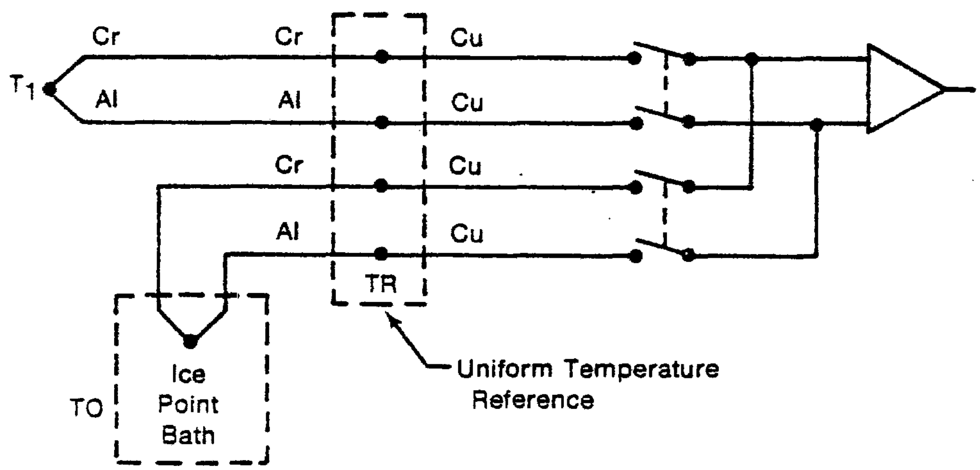

Figure 17 — Typical thermocouple channel

If several calibrated thermocouples are utilized to monitor the temperature of an ice point bath, statistically useful data can be recorded each time measurement data are recorded. Assuming that those thermocouple data are recorded and reduced to engineering units by processes identical to those employed for test temperature measurements, a stockpile of data will be gathered, from which data acquisition and reduction errors may be estimated.

For the purpose of illustration, suppose N calibrated Chromel-Alumel thermocouples are employed to monitor the ice bath temperature of a temperature measuring system similar to that depicted by figure 17. If each time measurement data are recorded, multiple scan recordings are made for each of the thermocouples, and if a multiple scan average (X_{ij}) is calculated for each thermocouple, then the average ($\overline{X}_j$) for all recordings of the jth thermocouple is

$$\overline{X}_j = \frac{\sum_{i=1}^{k_j} X_{ij}}{K_j} \tag{62}$$

where K_j is the number of multiple scan recordings for the jth thermocouple.

The grand average ($\overline{X}$) is computed for all monitor thermocouples as

$$\overline{X} = \frac{\sum_{j=1}^{N} \overline{X}_j}{N}. \tag{63}$$

The experimental standard deviation ($S_{\overline{x}}$) for the data acquisition and reduction processes is then

$$S_{\overline{x}} = \sqrt{\frac{\sum_{j=1}^{N}\sum_{i=1}^{K_j} (X_{ij} - \overline{X}_j)^2}{\sum_{j=1}^{N} (K_j - 1)}} \tag{64}$$

$$= 0.094 \text{ K (assumed for this example)}$$

The degrees of freedom associated with $S_{\overline{x}}$ are

$$\nu_{\overline{x}} = \sum_{j=1}^{N} (K_j - 1) \tag{65}$$

$$= 200 \text{ (assumed for this example)}$$

Data acquisition and reduction systematic error limits may be evaluated from the same ice bath temperature data if the temperature of the ice bath is continuously measured with a working standard such as a calibrated mercury-in-glass thermometer. There the systematic error limit is the largest observed difference between $\overline{X}$ and the temperature indicated by the working standard acquisition and reduction process. In this example, it is assumed to be 0.56°K, i.e.,

$$B_{\overline{x}} = 0.56°\text{K} \tag{66}$$

Error sources accounted for by this method are:

1) Ice point bath reference random error
2) Reference block temperature random error
3) Recording system resolution error
4) Recording system electrical noise error
5) Analog-to-digital conversion error
6) Chromel-Alumel thermocouple millivolt output vs. temperature curve-fit error
7) Computer resolution error

Several errors which are not included in the monitoring system statistics are:

1) Conduction error (B_C)
2) Radiation error (B_R)
3) Recovery error (B_Y)
4) Calibration error (B_1)

These errors are a function of probe design and environmental conditions. Detailed treatment of these error sources is beyond the scope of this work.

The experimental standard deviation for temperature measurements in this example is

$$S_\tau = \sqrt{S_1^2 + S_{\overline{x}}^2} \tag{67}$$

where

S_1 = calibration hierarchy experimental standard deviation

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$S_{\bar{x}}$ = data acquisition and reduction experimental standard deviation

$$S_\tau = \sqrt{0.056^2 + 0.094^2}$$

$$= 0.11\ ^\circ K$$

The degrees of freedom associated with S_τ are

$$\nu_\tau = \frac{(S_1^2 + S_2^2)^2}{\left(\frac{S_1^4}{\nu_1} + \frac{S_2^4}{\nu_2}\right)} \tag{68}$$

$$\nu_\tau = \frac{(0.056^2 + 0.094^2)^2}{\left(\frac{0.056^4}{53} + \frac{0.094^4}{200}\right)}$$

$$= 250 \text{ therefore } t_{95} = 2$$

Systematic error limits for the measurements are

$$B_\tau = \sqrt{B_1^2 + B_{\bar{x}}^2 + B_C^2 + B_R^2 + B_Y^2} \tag{69}$$

where

B_1 = calibration hierarchy systematic error limits

$B_{\bar{x}}$ = data acquisition and reduction systematic error limits

B_C = conduction error systematic error limits (negligible in this example)

B_R = radiation error systematic error limits (negligible in this example)

B_Y = recovery factor systematic error limits (negligible in this example)

$$B_\tau = \sqrt{0.578^2 + 0.56^2}$$

$$= 0.804^\circ K$$

Uncertainty for the temperature measurement is

$$U_\tau = (B_\tau + t_{95}\, S_\tau)$$

$$U_{99} = (B_\tau + t_{95}\, S_\tau),\ U_{95} = \sqrt{B_\tau^2 + (t_{95}\, S_\tau)^2}$$

$$U_{99} = (0.804 + 2 \times 0.11),\ U_{95} = \sqrt{0.804^2 + (2 \times 0.11)^2}$$

$$= 1.02^\circ K, \qquad = 0.83^\circ K \tag{70}$$

When ν is less than 30, t_{95} is determined from a Student's t table at the value of ν. Since ν_τ is greater than 30 here, use $t_{95} = 2$.

A.2.2.3 There are catalogs of discharge coefficients for a variety of venturis, nozzles and orifices. Cataloged values are the result of a large number of actual calibrations over a period of many years. Detailed engineering comparisons must be exercised to ensure that the flowmeter conforms to one of the groups tested before using the tabulated values for discharge coefficients and error tolerances.

To minimize the uncertainty in the discharge coefficient, it should be calibrated using primary standards in a recognized laboratory. Such a calibration will determine a value of A_{eff} = Ca and the associated systematic error limit and experimental standard deviation.

When an independent flowmeter is used to determine flowrates during a calibration for C, dimensional errors are effectively calibrated out. However, when C is calculated or taken from a standard reference, errors in the measurement of pipe and throat diameters will be reflected as systematic errors in the flow measurement.

Dimensional errors in large venturis, nozzles and orifices may be negligible. For example, an error of 0.001 inch in the throat diameter of a 5 inch critical flow nozzle will result in a 0.04% systematic error in airflow. However, these errors can be significant at large diameter ratios.

A.2.2.4 Non-ideal gas behavior and changes in gas composition are accounted for by selection of the proper values for compressibility factor (Z), molecular weight (M) and ratio of specific heats (γ) for the specific gas flow being measured.

When values of γ and Z are evaluated at the proper pressure and temperature conditions, airflow errors resulting from errors in γ and Z will be negligible.

For the specific case of airflow measurement, the main factor contributing to variation of composition is the moisture content of the air. Though small, the effect of a change in air density due to water vapor on airflow measurement should be evaluated in every measurement process.

A.2.2.5 The thermal expansion correction factor (F_a) corrects for changes in throat area caused by changes in flowmeter temperature.

For steels, a 17°K flowmeter temperature difference, between the time of a test and the time of calibration, will introduce an airflow error of 0.06% if no correction is made. If flowmeter skin temperature is determined to within 3°K and the correction factor applied, the resulting error in airflow will be negligible.

A.2.3 Propagation of error to airflow

For an example of propagation of errors in airflow measurement using a critical-flow venturi, consider a venturi having a throat diameter of 0.554 meters operating with dry air at an upstream total pressure of 88 126P_a and an upstream total temperature of 265.9°K.

Equation (71) is the flow equation to be analyzed:

$$W = \frac{\pi d^2}{4} C F_a \varphi^* \frac{P_1}{\sqrt{T_1}}$$

$$\varphi^* = \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{\gamma g M}{ZR}\right)} \qquad (71)$$

Assume, for this example, that the theoretical discharge coefficient (C) has been determined to be 0.995. Further assume that the thermal expansion correction factor (F_a) and the compressibility factor (Z) are equal to 1.0. Table 11 lists nominal values, systematic error limits, sample standard deviations and degrees of freedom for each error source in the above equation. (To illustrate the uncertainty methodology, we will assume a sample standard deviation of 0.000 5 in addition to a systematic error of 0.003.)

Note that, in table 11, airflow errors resulting from errors in F_a, Z, k, g, M and R are considered negligible.

Table 11 — Airflow measurement error sources

Error source	*Units*	*Nominal value*	*Systematic error limit*	*Experimental standard deviation*	*Degrees of freedom,* ν	*Uncertainty* U_{99}
P_1	P_a	88 126	277.02	126.79	96	530.60
T_1	K	265.9	0.8	0.11	250	1.02
d	m	0.554	2.54×10^{-5}	2.54×10^{-5}	100	7.62×10^{-5}
C		0.995	0.003	0.000 5	—	0.003
F_a		1.0	—	—	—	—
Z		1.0	—	—	—	—
γ		1.401	—	—	—	—
g		—	—	—	—	—
M	kg/kg-mole	28.95	—	—	—	—
R	J/K-kg-mole	8.314	—	—	—	—

From equation (71), airflow is calculated as

$$W = \frac{3.142}{4}(0.554)^2 \times 0.995 \times 1.0 \times \sqrt{\left(\frac{2}{2.401}\right)^{\frac{2.401}{0.401}} \left(\frac{1.401 \times 28.95}{8\,314}\right) \times \frac{88\,126}{\sqrt{265.9}}}$$

$$= 52.39 \text{ kg/sec.}$$

Taylor's series expansion of equation (71) with the assumptions indicated yields equations (72) and (73) from which the flow measurement experimental standard deviation and systematic error limits are calculated.

$$S_w = W\sqrt{\left(\frac{S_{P_1}}{P_1}\right)^2 + \left(\frac{-S_{T_1}}{2T_1}\right)^2 + \left(\frac{S_C}{C}\right)^2 + \left(\frac{2S_d}{d}\right)^2}$$

$$S_w = 52.39\left[\left(\frac{126.790}{88\,126}\right)^2 + \left(\frac{-0.11}{2 \times 265.9}\right)^2 + \left(\frac{0.000\,5}{995}\right)^2 + \left(\frac{2 \times 0.000\,025}{0.554}\right)^2\right]^{1/2}$$

$$= 52.39\sqrt{(0.001\,4)^2 + (-0.000\,2)^2 + (0.000\,503)^2 + (0.000\,09)^2}$$

$$= 0.078\,7 \text{ kg/sec} \tag{72}$$

$$B_w = W\sqrt{\left(\frac{B_{P_1}}{P_1}\right)^2 + \left(\frac{-B_{T_1}}{2T_1}\right)^2 + \left(\frac{B_C}{C}\right)^2 + \left(\frac{2B_d}{d}\right)^2}$$

$$B_w = 52.39\left[\left(\frac{277.02}{88\,126}\right)^2 + \left(\frac{-0.804}{531.8}\right)^2 + \left(\frac{0.003}{0.995}\right)^2 + \left(\frac{0.000\,05}{0.554}\right)^2\right]^{1/2}$$

$$= 52.39\sqrt{(0.003\,1)^2 + (-0.001\,5)^2 + (0.003\,0)^2 + (0.000\,09)^2}$$

$$= 0.241\,6 \text{ kg/seg} \tag{73}$$

By using the Welch-Satterthwaite formula, the degrees of freedom for the combined experimental standard deviation is determined from

$$\nu_w = \frac{\left[\left(\frac{\partial W}{\partial P_1}S_{P_1}\right)^2 + \left(\frac{\partial W}{\partial T_1}S_{T_1}\right)^2 + \left(\frac{\partial W}{\partial d}S_d\right)^2 + \left(\frac{\partial W}{\partial C}S_C\right)^2\right]^2}{\frac{\left(\frac{\partial W}{\partial P_1}S_{P_1}\right)^4}{\nu_{P_1}} + \frac{\left(\frac{\partial W}{\partial T_1}S_{T_1}\right)^4}{\nu_{T_1}} + \frac{\left(\frac{\partial W}{\partial d}S_4\right)^4}{\nu_d} + \frac{\left(\frac{\partial W}{\partial C}S_C\right)^4}{\nu_C}}$$

$$= \frac{\left[\left(\frac{S_{P_1}}{P_1}\right)^2 + \left(\frac{-S_{T_1}}{2T_1}\right)^2 + \left(\frac{2S_d}{d}\right)^2 + \left(\frac{S_C}{C}\right)^2\right]^2}{\frac{\left(\frac{S_{P_1}}{P_1}\right)^4}{\nu_{P_1}} + \frac{\left(\frac{-S_{T_1}}{2T_1}\right)^4}{\nu_{T_1}} + \frac{\left(\frac{2S_d}{d}\right)^4}{\nu_d} + \frac{\left(\frac{S_C}{C}\right)^4}{\nu_C}} \tag{74}$$

which results in an overall degrees of freedom > 30, and, therefore, a value of t_{95} of 2.0.

Total airflow uncertainty is then,

$$U_{99} = (B_w + t_{95}\,S_w),\ U_{95} = \sqrt{B_w^2 + (t_{95}\,S_w)^2}$$

$$U_{99} = [0.241\,6 + 2 \times 0.078\,7]$$

$$= 0.40 \text{ kg/sec}$$

$$= 0.8\%$$

$$U_{95} = 0.29 \text{ kg/sec}$$

$$= 0.55\% \tag{75}$$

A.3 Example two — comparative test

A.3.1 Definition of the measurement process

The objective of a comparative test is to determine with the smallest measurement uncertainty the net effect of a design change, such as a new part. The first test is performed with the standard or baseline configuration. A second test, identical to the first except that the design change is substituted in the baseline configuration, is then carried out. The difference between the measurement results of the two tests is an indication of the effect of the design change.

As long as we only consider the difference or net effect between the two tests, all the fixed, constant, systematic errors will cancel out. The measurement uncertainty is composed of random errors only.

For example, assume we are testing the effect on the gasflow of a centrifugal compressor from a change to the inlet inducer. At constant inlet and discharge conditions, and constant rotational speed, will the gas flow increase? If we test the compressor with the old and new inducers and take the difference in measured airflow as our defined measurement process, we obtain the smallest uncertainty. All the systematic errors cancel. Note that, although the comparative test provides an accurate net effect, the absolute value (gasflow with the new inducer) is not determined or if calculated, as in example one, it will be inflated by the systematic errors. Also, the small uncertainty of the comparative test can be significantly reduced by repeating it several times.

A.3.2 Measurement error sources

All errors result from random errors in data acquisition and data reduction. Systematic errors are effectively zero. Random error values are identical to those in example one, except that calibration random errors become systematic errors and, hence, effectively zero.

A.3.2.1 Comparative tests shall use the same test facility and instrumentation for each test. All calibration errors are systematic and cancel out in taking the difference between the test results.

$$B_1 = 0$$

and

$$S_1 = 0, S_C = 0$$

A.3.2.2

$$S_p = S_2$$

$$= 119.039 \text{ Pa} \qquad \text{(see equation (47))}$$

$$\nu_p = \nu_2 = 77 \qquad \text{(see equation (48))}$$

$$S_\tau = S_{\bar{x}}$$

$$= 0.094°\text{K} \qquad \text{(see equation (64))}$$

$$\nu_\tau = \nu_{\bar{x}}$$

$$= 200 \qquad \text{(see equation (65))}$$

A.3.2.3 The test result is the difference in flow between two tests.

$$\Delta_w = W_1 - W_2$$

$$S_{\Delta w} = \sqrt{S_{w1}^2 + (-1)^2 S_{w2}^2} = S_w \sqrt{2}$$

$$U_{\Delta w_{99}} = (B_{\Delta w} + 2S_{\Delta w}) \qquad U_{\Delta w_{95}} = \sqrt{(B_{\Delta w})^2 + (2S_{\Delta w})^2}$$

$$= (0 + 2S_{\Delta w}) \qquad = \sqrt{0^2 + (2S_{\Delta w})^2}$$

$$= 2S_{\Delta w} \qquad = 2S_{\Delta w}$$

$$U_{\Delta w_{99}} = 2S_w \sqrt{2} \qquad U_{\Delta w_{95}} = 2S_w \sqrt{2}$$

$$S = \pm 52.39 \left[\left(\frac{119.037}{88\,126} \right)^2 + \left(\frac{-0.094}{2 \times 265.9} \right)^2 + \left(\frac{0.000\,5}{0.995} \right)^2 + \left(\frac{0.000\,05}{0.554} \right)^2 \right]^{1/2}$$

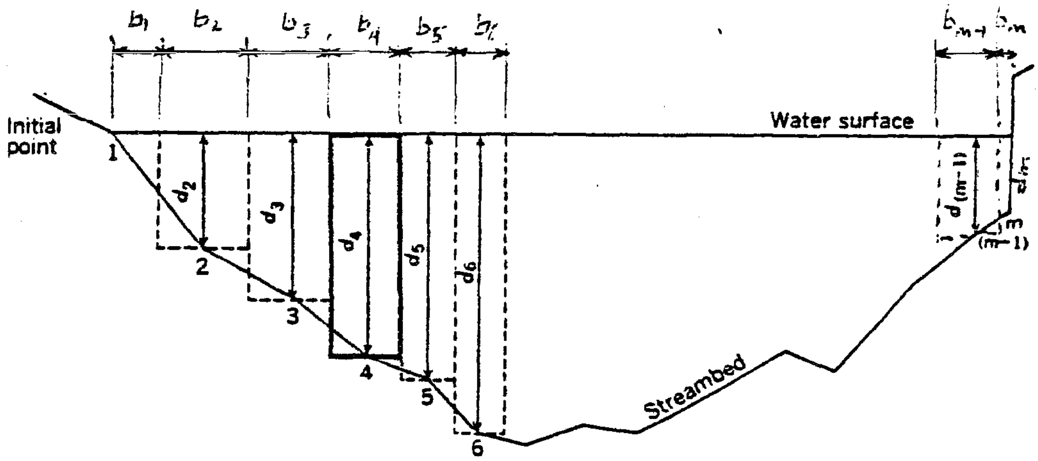
$$S_w = 0.076\,2 \text{ kg/sec} \qquad S_{\Delta w} = 0.107\,8 \text{ kg/sec}$$

$$U_{\Delta w_{99}} = 0.215\,5 \text{ kg/sec} \qquad U_{\Delta w_{95}} = 0.215\,5 \text{ kg/sec}$$

$$= 0.41\% \qquad = 0.41\%$$

(see equation (75))

A.3.2.4 Note that the differences shown in table 12 are entirely due to differences in the measurement process definitions. The same fluid flow measurement system might be used in both examples. The comparative test has the smallest measurement uncertainty, but this uncertainty value does not apply to the measurement of absolute level of fluid flow, only to the difference.



Explanation

1, 2, 3, ... m	Observation points
$b_1, b_2, b_3, \ldots b_m$	Breadth (metres) of segment associated with the observation point
$d_1, d_2, d_3, \ldots d_m$	Depth of water (metres) at the observation point
Dashed lines	Boundary of segments: one heavily outlined

If x and y are respectively horizontal and vertical coordinates of all the points in the cross-section, and A is its total area, then the precise mathematical expression for q_v, the true volumetric flowrate (discharge) across the area, can be written as

Figure 18 — Definition sketch of velocity-area method of discharge measurement (midsection method)

Table 12 — Error comparisons of examples one and two

	Example One — Facility	*Example Two — Facility*
1. Experimental standard deviation, kg_m/sec (s)	0.078 7	0.076 2
2. Degrees of freedom (v)	>30	>30
3. Systematic error, kg_m/sec (B)	0.245 7	0
4. Uncertainty, kg_m/sec	0.40	0.22

Annex B — Examples on estimating uncertainty in open channel flow measurement

B.1 General

Evaluation of the overall uncertainty of a flow in an open channel will be demonstrated by considering (1) the velocity-area method and (2) the weirs method.

The method of measuring the flow is such that it is impractical to eliminate interdependent variables from the equation before estimating flow uncertainty. Therefore, it involves evaluation of the interdependent uncertainties specified in 7.4. In addition, measurement conditions often make it impossible to obtain the replicate measurements needed for evaluation of experimental standard deviations. Thus, it is desirable to express the random errors as well as the systematic errors as error limits. Under these conditions, it also is appropriate to assume that all the random error limits are equivalent to two experimental standard deviations. Under this assumption, the random error limits can be propagated with each other by means of the same root-sum-square formulas as the systematic error limits (see equations 19-22).

B.2 Example one — velocity area method

B.2.1 The equation for discharge in an open channel — velocity area

The channel cross-section under consideration is divided into segments by m verticals. The breadth, depth and mean velocity associated with any vertical i are denoted by b_i, d_i and $\bar{v}_i$ respectively. (see figure 18) The product $Q_i = b_i d_i \bar{v}_i$ represents an approximation to the discharge (volumetric flow rate) in the i-th segment. The sum over all segments,

$$Q_{vo} = \sum_{i=1}^{m} Q_i = \sum_{i=1}^{m} b_i d_i \bar{v}_i \tag{76}$$

represents an estimated or observed value of the total discharge.

If x and y are respectively horizontal and vertical coordinates of all the points in the cross-section, and A is its total area, then the precise mathematical expression for Q_v, the true volumetric flowrate (discharge) across the area, can be written as

$$Q_v = \iint_A v(x,y)\,dx\,dy \tag{77}$$

The true discharge and the observed discharge are related by a proportionality factor representing the approximation of the integral equation (77) by the finite sum equation (76), thus:

$$Q_v = F_m Q_{vo} = F_m \sum_{i=1}^{m} b_i d_i \bar{v}_i \tag{78}$$

where

$$F_m = \left[\iint_A v(x,y)\,dx\,dy\right] \Big/ \left[\sum_{i=1}^{m} b_i d_i \bar{v}_i\right]$$

In practice, F_m can be evaluated from analysis of measurements in which m is sufficiently large for the effects on Q_{vo} of omitting verticals, in stages, to be determined. F_m is subject to a random uncertainty.

It may be convenient in practice to take an F_m variation with m that is a mean value of values for sections of several different rivers, taken together. Then the actual variations of F_m from river to river, as compared with the meaned variation, will involve both systematic and random errors.

F_m is dependent on the number of verticals m, and tends to unity as m increases without limit. Thus, equation 78 can be written approximately as

$$Q_v = \sum_{i=1}^{m} (b_i d_i \bar{v}_i) \tag{79}$$

with increasing accuracy as m increases.

This last form is the one that is given in ISO 748.

B.2.2 The overall uncertainty of the flow determination

It is plausible to assume that, at a given m, F and Q_{vo} can be treated as independent variables.

However, the Q_i in principle are not independent of one another, since the value corresponding to any one vertical will be related to the values of adjacent verticals. Furthermore, there is an interdependence between the d_i and $\bar{v}_i$ corresponding to any particular vertical. Thus, applying the principles for combining random errors (see clause 5) and denoting random error by S, the following expression for S_Q, the uncertainty of Q, can be derived from equation 78.

$$\left[\frac{S_Q}{Q_v}\right]^2 = \left[\frac{S_{F_m}}{F_m}\right]^2 + \sum_{i=1}^{m}\left(\frac{Q_i}{Q_{vo}}\right)^2 \left\{\left[\frac{S_{b_i}}{b_i}\right]^2+\left[\frac{S_{d_i}}{d_i}\right]^2+\left[\frac{S_{\bar{v}_i}}{\bar{v}_i}\right]^2\right\} + \frac{2}{Q_{vo}^2}\left\{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} S_{i,j} + \sum_{i=1}^{m}\left[\left(\frac{Q_i^2}{d_i\bar{v}_i}\right)S_{d,\bar{v}_i}\,\bar{v}_i\right]\right\} \qquad (80)$$

where S_{ij} arise from the interdependence between Q_i and Q_j and $S_{d\bar{v}_i}$ from the interdependence between d_i and $\bar{v}_i$.

It is convenient to introduce the notation S′ for relative random error.

Thus S_{b_i}/b_i is written S'_{b_i}, S_{F_m}/F_m is written S'_{F_m}, and, neglecting S_{ij}, and $S_{d\bar{v}_i}$, equation (80) becomes

$$S'^2_Q = S'^2_{F_m} + \sum_{i=1}^{m} \frac{Q_i}{Q_{vo}} \left(S'^2_{b_i} + S'^2_{d_i} + S'^2_{\bar{v}_i} \right)$$

If the relative errors S'_b are all nearly enough equal, of value S'_{b_i}, and similarly for the S'_{d_i} and $S'_{d\bar{v}_i}$, then

$$S'^2_Q = S'^2_{F_m} + \left(S'^2_b + S'^2_d + S'^2_{\bar{v}} \right) \sum_{i=1}^{m} \left(Q_i/Q_{vo} \right)^2 \qquad (81)$$

If the verticals are so located that $Q_i \approx Q_{vo}/m$, then

$$S'^2_Q \approx S'^2_{F_m} + \frac{1}{m}\left(S'^2_b + S'^2_d + S'^2_{\bar{v}} \right) \qquad (82)$$

In multi-point velocity-area methods, velocity is measured at several points on a vertical, and the mean value is obtained by graphical integration or as a weighted average. The latter treatment can be expressed mathematically for a particular value as

$$\bar{v} = \sum_{p=1}^{k} w_p v_p$$

where the w_p are constant weighting factors. The suffix i that identifies the particular vertical is omitted to simplify the symbolism. The points usually are chosen so that $\Sigma\, w_p = 1$. This equation can also represent the single-point method, by taking $k = 1$.

In all cases, the estimates $\bar{v}$ so computed are subject to errors. These errors are due to improper placement of the meter at depth and to deviations of the actual velocity profile from the presumed profile. The effect of these errors can be expressed by means of a multiplicative coefficient P analogous to the coefficient F_m used for similar purposes in equation (78). The same analysis that led to equation (80) then yields the following expression for relative random error of the average velocity $\bar{v}$:

$$S'^2_{\bar{v}} = S'^2_p + S'^2_v \frac{\Sigma\,(w_p\, v_p)^2}{(\Sigma\, w_p\, v_p)^2}$$

in which S′ denotes relative random error in the subscript variable, v is measured point velocity, and the ratio of wv-sums expresses the variability of weighted velocity over the depth of the vertical. For a uniform k-point velocity profile, this ratio would equal 1/k. For an extremely non-uniform profile, in which a single term dominated all the others, the ratio would equal 1. The latter value is adopted, at least for small k values, for the sake of conservatism, with the result

$$S'^2_{\bar{v}} = S'^2_p + S'^2_v$$

This choice also helps to represent the effect of any unaccounted-for correlations among point-velocity errors in the same vertical.

In practice, the random error in the velocity measurement at a point is assumed to be due to a meter-calibration random relative error, S'_c, together with a stream pulsation random error S'_e. Then the random relative error for point velocities is

$$S'^2_v = S'^2_c + S'^2_e$$

The corresponding random relative error for average velocity in the vertical is

$$S'^2_{\bar{v}} = S'^2_p + S'^2_c + S'^2_e \qquad (83)$$

B.2.3 Calculation of uncertainty

It is required to calculate the uncertainty in a current-meter gauging from the following particulars:

Number of verticals used	20
Exposure time of current meter at each point in the vertical	3 min
Number of points taken in the vertical (single point, two points, etc.)	2
Type of current meter rating (individual or group)	individual
Average velocity in measuring section	above 0.3 m/s

Details of procedure are described in ISO 748.

The random and systematic errors are combined by the root-sum-square method as stated in 8.3, i.e., if S'_Q and B'_Q are the percentage overall random and systematic relative errors respectively, then U'_Q, the percentage uncertainty in the current meter gauging, is

$$U'_{Q_{95}} = \sqrt{(2S'_Q)^2 + B'^2_Q} \text{ and } U'_{Q_{99}} = B'_Q + 2S'_Q$$

B.2.3.1 The error equation used for evaluating the overall random error is (see equation (82).)

$$S'_Q = \sqrt{S'^2_m + \frac{1}{m}\left(S'^2_b + S'^2_d + S'^2_{\bar{v}}\right)}$$

where

S'_Q is the overall percentage random error

S'_m is the percentage random error due to the limited number of verticals used;

S'_b is the percentage random error in measuring width of segments;

S'_d is the percentage random error in measuring depth of segments;

$S'_{\bar{v}}$ is the percentage random error in estimating the average velocity in each vertical

$$S'_{\bar{v}} = \pm\sqrt{S'^2_p + S'^2_c + S'^2_e}$$

(see equation (85))

where

S'_p is the percentage error due to limited number of points taken in the vertical (in the present example the two-point method was used, i.e., at 0.2 and 0.8 from the surface respectively);

S'_c is the percentage error of the current meter rating (in the present example an individual rating was used at velocities of the order of 0.30 m/s);

S'_e is the percentage error due to pulsations (error due to the random fluctuation of velocity with time; the time of exposure in the present example was three one-minute readings of velocity.)

The percentage values of the above partial errors at the 95% confidence level are tabulated in B.2.3.2.

The equation for calculating the overall systematic error is

$$B'_Q = \sqrt{B'^2_b + B'^2_d + B'^2_c}$$

where

B'_Q is the overall percentage systematic uncertainty in discharge;

B'_b is the percentage systematic error in the instrument measuring width;

B'_c is the percentage systematic error in the instrument measuring depth; and

B'_d is the percentage systematic error in the current meter rating tank.

The systematic errors in the current meter gauging are confined to the instruments measuring width, depth and velocity and should be restricted to 1% as shown in B.2.3.2.

B.2.3.2 The values of the error elements affecting uncertainty in discharge are tabluated below as percentage errors at the 95% confidence level. The numerical values are taken from ISO 748. It is recommended, however, that each user determine independently the values of the errors for any particular measurement.

Table 13 — Error elements affecting uncertainty in discharge

Error source	*Units*	*(2S') random error limit (2S:95%)*	*(B') percentage systematic error limit*
F_m, number of verticals	—	5.0	—
b, segment width	m	0.5	1.0
d, segment depth	m	0.5	1.0
v_p, number of profile points	m/s	7.0	—
v_c, meter calibration	m/s	2.0	1.0
v_e, meter exposure time	m/s	10.0	—

Then, the overall random error in discharge is given by

$$2S'_Q = 2\sqrt{S'^2_{F_m} + \frac{1}{m}(S'^2_b + S'^2_d + S'^2_p + S'^2_c + S'^2_e)}$$

$$= \sqrt{25 + \frac{1}{20}(0.25 + 0.25 + 49 + 4 + 100)}$$

$$= 5.7\%$$

The overall systematic error is

$$B'_Q = \sqrt{1^2 + 1^2 + 1^2}$$

$$= 1.7\%$$

The combination of both random and systematic errors then gives the overall percentage uncertainty in discharge, U'_Q.

$$U'_{Q_{95}} = \sqrt{(2S'_Q)^2 + B'^2_Q} \qquad U'_{Q_{99}} = B'_Q + 2S'_Q$$

$$= \sqrt{5.7^2 + 1.7^2} \qquad = 1.7 + 5.7$$

$$= 5.9\% \qquad = 7.4\%$$

B.2.3.3 The discharge measurement may be expressed in the following form:

Discharge	(Q) m^3/s
(Combined) uncertainty, $U'_{Q_{95}}$	5.9%
(Combined) uncertainty, $U'_{Q_{99}}$	7.4%
Random error ($2S'_Q$)	5.7%
Systematic error (B'_Q)	1.7%

Uncertainties calculated in accordance with ISO 5168.

B.3 Example two — weir measurement

B.3.1 Weir data

It is required to calculate the discharge and the uncertainty in discharge for a triangular profile weir given the following details: (see figure 19)

Gauged head, h	0.67m
Breadth of weir, b	10m
Crest height, P	1m
Coefficient of discharge, Cd	1.163
Coefficient of velocity, Cv	1.054

The discharge equation is

$$Q = (2/3)^{3/2}\, C_d C_v \sqrt{g}\, b\, h^{3/2} \tag{84}$$

Details of the procedure are described in ISO 4360.

B.3.2 Uncertainty equations

Taylor series analysis of the discharge equation yields the following uncertainty equations, which can be used for both random and systematic errors:

$$S'_Q = \sqrt{S'^2_{c_d c_v} + S'^2_b + (3/2)^2 S'^2_h}$$

and

$$B'_Q = \sqrt{B'^2_{c_dc_v} + B'^2_b + (3/2)^2 B'^2_h} \tag{85}$$

in which S′ and B′ denote percentage errors of the subscript variables.

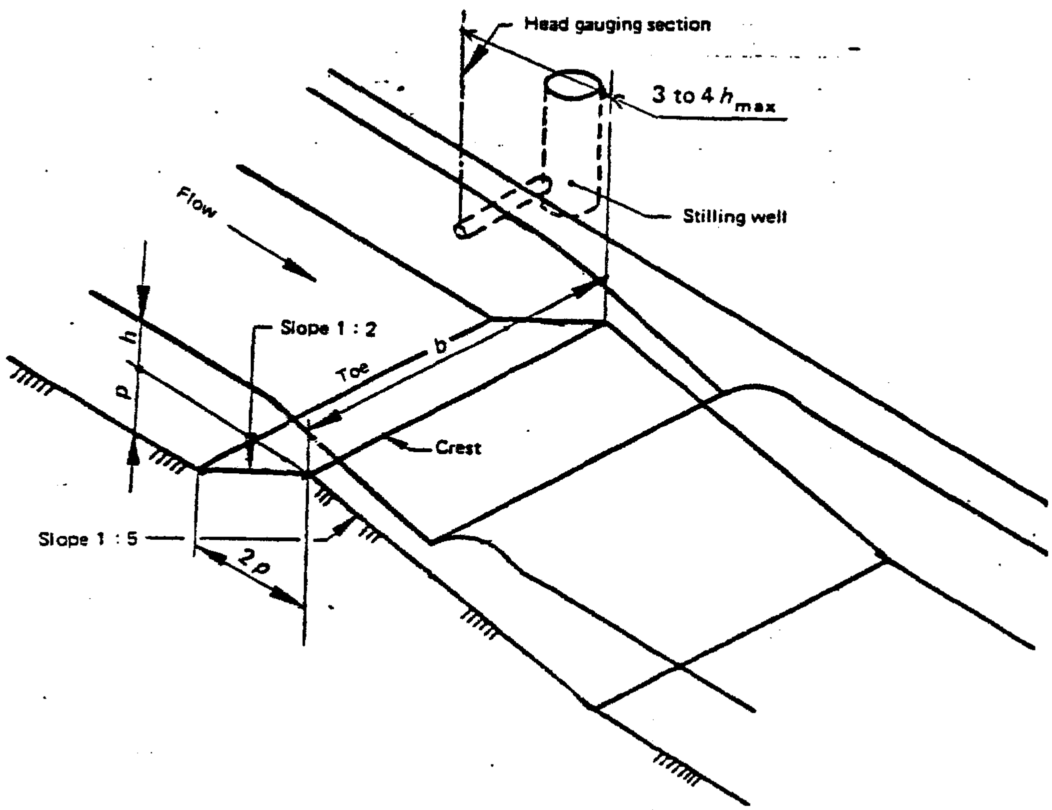


Figure 19 — Triangular profile weir

B.3.3 Evaluation of discharge and uncertainties

The values of the error elements affecting this problem are tabulated below as error limits at the 95% confidence level. The numerical values are based on information given in ISO 4360. It is recommended, however, that each user determine independently the values of the errors for any particular measurement. (See table 14)

Table 14 — Error element values

Variable	*Units*	*Nominal value*	*(2S′) random error limit (2S:95%)*	*(B′) systematic error limit*
h	m	0.67	0.003 0.45%	0.003 0.45%
b	m	10.00	0.	0.01 0.1%
C_dC_v	—	1.226	0.5%	1.5%
g	m/s^2	9.81	0.	0.

0260r

Substitution of the nominal values into the discharge equation yields

$$Q = (3/2)^{3/2} \times (1.226) \times \sqrt{9.81} \times 10 \times (0.67)^{3/2}$$

$$= 11.46 \text{ m}^3/\text{s}$$

Evaluation of the random errors yields

$$2S'_Q = \sqrt{(0.5)^2 + (3/2)^2 (0.45)^2}$$

$$= 0.84\%$$

Evaluation of the systematic errors yields

$$B'_Q = \sqrt{(1.5)^2 + (0.1)^2 + (3/2)^2 (0.45)^2}$$

$$= 1.65\%$$

Combining the random and systematic errors by the root-sum-square (RSS) method yields

$$U'_{Q_{95}} = \sqrt{(2S'^2_Q) + B'^2_Q} \qquad U'_{Q_{99}} = B'_Q + 2S'_Q$$

$$= \sqrt{(0.84)^2 + (1.65)^2} \qquad = 1.65 + 0.84$$

$$= 1.85\% \qquad = 2.49\%$$

B.3.4 Presentation of results

The discharge Q may be reported as follows:

Discharge	6m³/s
(Combined) uncertainty, $U'_{Q_{95}}$	%
(Combined) uncertainty, $U'_{Q_{99}}$	2.5%
Random error ($2S'_Q$)	0.8%
Systematic error (B'_Q)	1.6%

Uncertainties calculated in accordance with ISO 5168.

Annex C — Small sample methods

C.1 Student's t.

When the experimental standard deviation is based on small samples ($N \leq 30$), uncertainty is defined as:

$$U_{ADD} = B + t_{95}S \tag{86}$$

$$U_{RSS} = \sqrt{B^2 + (t_{95}S)^2} \tag{87}$$

For these small samples, the interval $[\overline{X} - t_{95}S/\sqrt{N}, \overline{X} + t_{95}S/\sqrt{N}]$ will contain the true unknown average, μ, 95% of the time. If the systematic error is negligible, this statistical confidence interval is the uncertainty interval. t_{95} is the 95th percentile point for the two-tailed Students t-distribution. For small samples, t will be large, and for larger samples t will be smaller, approaching 1.96 as a lower limit. The t-value is a function of the number of degrees of freedom (ν) used in calculating S. Since 30 degrees of freedom (ν) yield a t of 2.05 and infinite degrees of freedom yield a t of 1.96, an arbitrary selection of t = 2 is used for simplicity for values of ν from 30 to infinity. See table 15.

C.2 Degrees of freedom for small samples

In a sample, the number of degrees of freedom (ν) is the sample size, N. When a statistic is calculated from the sample, the degrees of freedom associated with the statistic is reduced by one for every estimated parameter used in calculating the statistic. For example, from a sample of size N, $\overline{X}$ is calculated and has N degrees of freedom, and the experimental standard deviation, S, is calculated using equation (1), and has N-1 degrees of freedom because $\overline{X}$ is used to calculate S. In calculating other statistics, more than one degree of freedom may be lost. For example, in calculating the standard error of a curve fit, the number of degrees of freedom which are lost is equal to the number of estimated coefficients for the curve, N − 2.

When all random error sources have large sample sizes (i.e., $\nu_{ij} > 30$) the calculation of is unnecessary and 2 is substituted for t_{95}. However, for small samples, when combining experimental standard deviations by the root-sum-square method (see equation (20) for example), the degrees of freedom (ν) associated with the combined experimental standard deviations is calculated using the Welch-Satterthwaite formula (88).

For example: the degrees of freedom for the calibration experimental standard deviation (S_1) given by equation (20), is:

$$\nu_1 = \frac{\left(\sum_{i=1}^{4} S_{i1}^2\right)^2}{\sum_{i=1}^{4} \frac{S_{i1}^4}{\nu_{i1}}} = \frac{(S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2)^2}{\frac{S_{11}^4}{\nu_{11}} + \frac{S_{21}^4}{\nu_{21}} + \frac{S_{31}^4}{\nu_{31}} + \frac{S_{41}^4}{\nu_{41}}} \quad (88)$$

where ν_{i1} is the degrees of freedom of each elemental experimental standard deviation in the calibration process.

The degrees of freedom for the measurement experimental standard deviation (S), as given by equation (21) is:

$$\nu = \frac{\left(\sum_{j=1}^{3} \sum_{i=1}^{K} S_{ij}^2\right)^2}{\sum_{j=1}^{3} \sum_{i=1}^{K} \frac{S_{ij}^4}{\nu_{ij}}} \quad (89)$$

If the test result is an average, $\bar{X}$, based on a sample of size N,

$$S_{\bar{x}} = \frac{S_X}{\sqrt{N}} \quad (90)$$

As $\sqrt{N}$ is a known constant, the degrees of freedom of $S_{\bar{x}}$ is the same as S, i.e.

$$\nu_{S_{\bar{x}}} = \nu \quad (91)$$

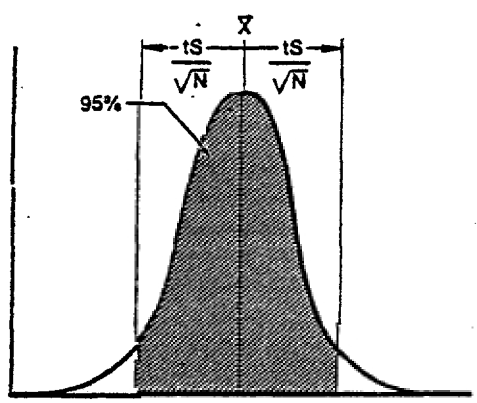
Table 15 — Two-tailed student's "t" table

SMALL SAMPLE METHODS

Degrees of freedom < 30

Degrees of freedom	t_{95}	Degrees of freedom	t_{95}
1	12.706	17	2.110
2	4.303	18	2.101
3	3.182	19	2.093
4	2.776	20	2.086
5	2.571	21	2.080
6	2.447	22	2.074
7	2.365	23	2.069
8	2.306	24	2.064
9	2.262	25	2.060
10	2.228	26	2.056
11	2.201	27	2.052
12	2.179	28	2.048
13	2.160	29	2.045
14	2.145	∞	1.96
15	2.131		
16	2.120		

Two-tailed student's "t" table



30 or more use 2.0

C.3 Propagating the degrees of freedom

The Student's t value of table 16 to be used in calculating the uncertainty of the test result (equations (86) or (87)) is based on ν_r, the degrees of freedom of S_r. If the degrees of freedom of any measurement standard deviation is less than 30, the degrees of freedom of the result also may be less than 30. In such cases, the following small sample method may be used to determine ν_r. This is defined for the absolute experimental standard deviation according to the Welch-Satterthwaite formula by:

$$\nu_r = \frac{S_r^4}{\sum_{i=1}^{J} \frac{(\theta_i S_{\bar{P}_i})^4}{\nu_{\bar{P}_i}}} \quad (92)$$

and for the relative experimental standard deviation by:

$$\nu_r = \frac{(S_r/r)^4}{\sum_{i=1}^{J} \frac{(\theta_i S_{\bar{P}_i}/\bar{P}_i)^4}{\nu_{\bar{P}_i}}} \tag{93}$$

where

$$S_r = \sqrt{\Sigma(\theta_i S_i)^2}$$

and the degrees of freedom of the experimental standard deviation (S_{P_i}) of the independent measurements is usually given by:

$$\nu_{\bar{P}_i} = (N_i - 1) \tag{94}$$

NOTE: The degrees of freedom for the relative and absolute experimental standard deviations are identical.

Welch-Satterthwaite degrees of freedom may contain fractional, decimal parts. The fractions should be dropped or truncated as rounding down is conservative with Student's t, i.e. $\nu = 13.6$ should be treated as $\nu = 13.0$.

Annex D — Outlier treatment

D.1 General

All measurement systems may produce spurious data points. These points may be caused by temporary or intermittent malfunctions of the measurement system or they may represent actual variations in the measurement. Errors of this type should not be included as part of the uncertainty of the measurement. Such points are meaningless as test data. They should be discarded. Figure 20 shows a spurious data point called an outlier.

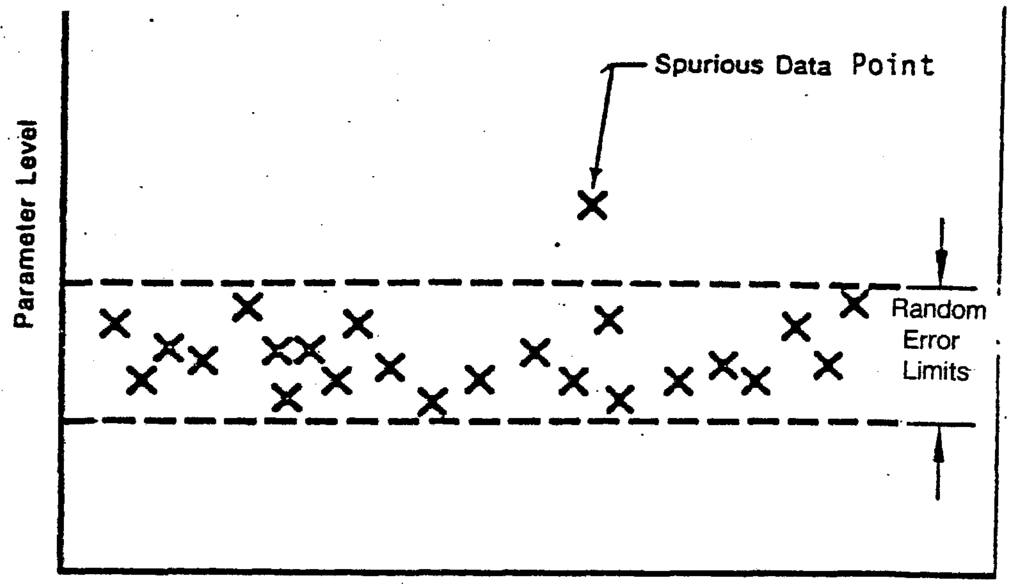


Figure 20 — Outlier outside the range of acceptable data

All data should be inspected for spurious data points as a continuing check on the measurement process. Points should be rejected based on engineering analysis of instrumentation, thermodynamics, flow profiles and past history with similar data. To ease the burden of scanning large masses of data, computerized routines are available to scan steady-state data and flag suspected outliers. The flagged points should then be subjected to an engineering analysis.

The effect of these outliers is to increase the random error of the system. A test is needed to determine if a particular point from a sample is an outlier. The test should consider two types of errors in detecting outliers:

(1) Rejecting a good data point
(2) Not rejecting a bad data point

The probability for rejecting a good point is usually set at 5%. This means that the odds of rejecting a good point are 20 to 1 (or less). The odds will be increased by setting the probability of (1) lower. However, this practice decreases the probability of rejecting bad data points. The probability of rejecting a good point will require that the rejected points be further from the calculated mean and fewer bad data points will be identified. For large sample sizes, several hundred measurements, almost all bad data points can be identified. For small samples (five or ten), bad data points are hard to identify.

One test in common usage for determining whether spurious data are outliers is Grubbs' Method.

D.2 Grubbs' method

Consider a sample (X_i) of N measurements. The mean ($\overline{X}$) and an experimental standard deviation (S) are calculated by equation (1). Suppose that (X_j), the j-th observation, is the suspected outlier; then, the absolute statistic calculated is:

$$T_n = \left[\frac{X_j - \overline{X}}{S} \right]$$

Using table 16, a value of T_n is obtained for the sample size (N) and the 5% significance level (P). This limits the probability of rejecting a good point to 5%. (The probability of not rejecting a bad data point is not fixed. It will vary as a function of sample size.

The test for the outlier is to compare the calculated T_n with the table T_n.

If T_n calculated is larger than or equal to T_n table, we call X_j an outlier.

If T_n calculated is smaller than T_n table, we say X_j is not an outlier.

Table 16 — Rejection values for Grubbs' method

Sample size N	*5% (1-sided)*	*Sample size*	*5% (1-sided)*
3	1.150	20	2.56
4	1.46	21	2.58
5	1.67	22	2.60
6	1.82	23	2.62
7	1.94	24	2.64
8	2.03	25	2.66
9	2.11	30	2.75
10	2.18	35	2.82
11	2.23	40	2.87
12	2.29	45	2.92
13	2.33	50	2.96
14	2.37	60	3.03
15	2.41	70	3.09
16	2.44	80	3.14
17	2.47	90	3.18
18	2.50	100	3.21
19	2.53		

D.3 Example

In the following sample of 40 values,

26	79	58	24	1	−103	−121	−220
−11	−137	120	124	129	−38	25	−60
−148	−52	−216	−12	−56	89	8	−29
−107	20	9	−40	40	2	10	166
126	−72	179	41	127	−35	<u>334</u>	<u>−555</u>

suspected outliers are 334 and -555 (underlined).

To illustrate the calculations for determining whether -555 is an outlier from figure 21.

Mean ($\overline{X}$) = 1.125
Exp. Std. Dev. = 140.813 6
Sample Size = 40

$$T_{n_{calc}} = \frac{-555 - 1.125}{140.813\ 6} = 3.95$$

from table 16 using Grubbs' Method for N = 40 @ 5% level of significance (one-sided),

$$T = 2.87$$

Therefore, since 3.95 > 2.87

$$(T_{n_{calc}}) > (T_{n_{table}})$$

−555 is an outlier according to Grubbs' test.

Suspected outlier	Calculated Tn	Table Tn P=5	Sample size (N)	Experimental standard deviation(s)	Mean X
−555	3.95	2.87	40	140.8	1.125
334	2.91 (stop)	2.86	39	109.6	15.385
−220	2.33	2.85	38	97.5	7.000

Figure 21 is a normal probability plot of this data with the suspected outliers indicated. In this case, the engineering analysis indicated that the −555 and 334 readings were outliers, agreeing with the Grubbs' test results.

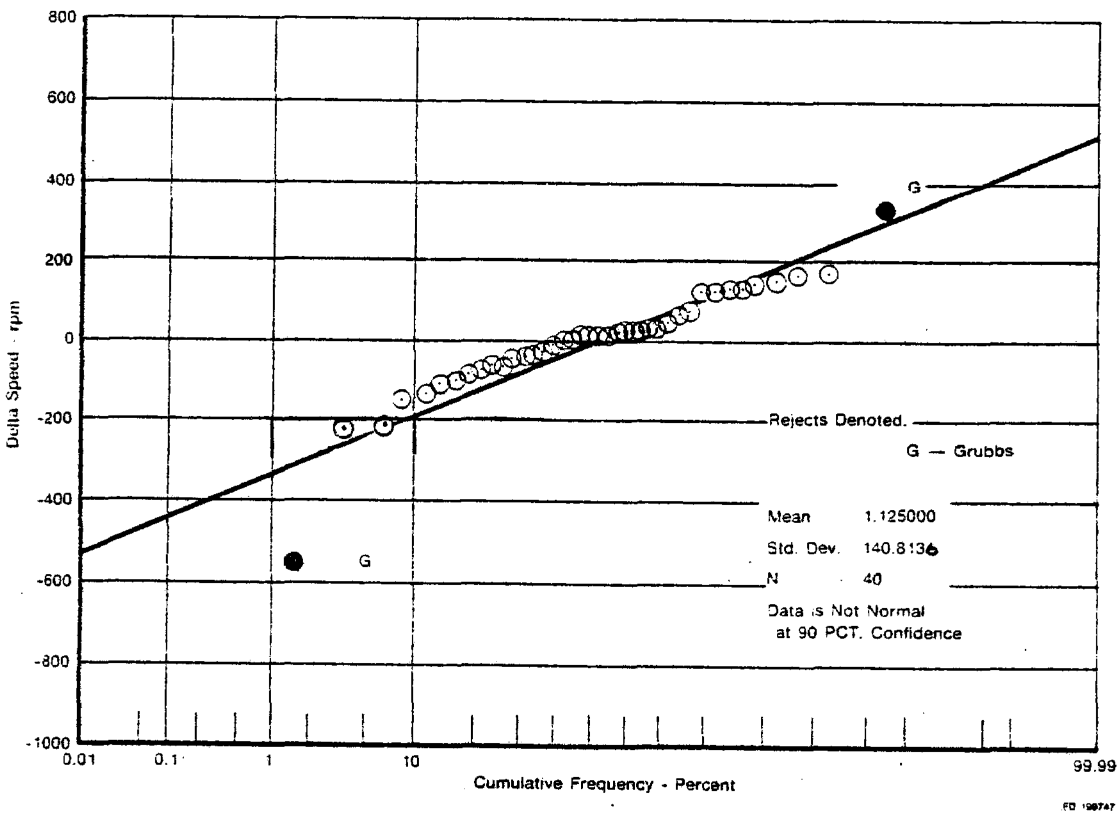


Figure 21 — Results of outlier tests

Annex E — Statistical uncertainty intervals

It is usually impossible to determine the statistical distribution of the systematic errors (β) because they are usually subjective judgments, i.e. not based on data. However, if there is information to justify a distribution assumption, it is possible to use rigorous statistical methods to calculate the uncertainty interval. The validity of this assumption must be left to the judgement of the reader. The purpose of this annex is to describe the methods, given the assumption.

E.1 Assumed systematic error distribution

If it is assumed that the systematic errors (B) are actually the maximum possible upper and lower limit of the true, unknown systematic error (β), and that β is equally probable anywhere within the limits, then the standard deviation of the systematic error may be determined by

$$\sigma_B = \frac{B}{\sqrt{3}} \tag{95}$$

As depicted in figure 22.

The validity of this assumption cannot be proved or disproved. It is a matter of judgement.

E.2 U_{RSS}

The systematic error limit of the measurement result may be calculated as before

$$B = \sqrt{\sum_{i}^{N} (\theta_i B_i)^2} \tag{96}$$

The experimental standard deviation of the systematic error is estimated as:

$$S_B = \frac{B}{\sqrt{3}} \tag{97}$$

The uncertainty is

$$U_{RSS} = \sqrt{(1.645\ S_B)^2 + (2S)^2} \tag{98}$$

for large samples, where S is the experimental standard deviation of the random error.

Assuming there are many sources of systematic and random errors, say ten or more, the Central Limit Theorem states that sums of samples taken from any distribution(s) will tend toward normality. Therefore, the true error (δ) should be distributed as a normal distribution with standard deviation equal to the root-sum-square of the systematic and random error experimental standard deviations. This will be illustrated in E.4. If small samples are used to estimate the random error experimental standard deviations, Student's t and the Welch-Satterthwaite approximation will be needed as described in annex C.

E.3 U_{ADD}

With the additive model of uncertainty, the assumed distribution does not affect the answer. The systematic error, B, is still determined as equation (96) and there is no advantage to calculating a standard deviation of systematic error.

$$U_{ADD} = B + t_{95}S \tag{99}$$

E.4 Monte Carlo example

To illustrate the Central Limit Theorem, the sum of a random sample from each of the ten rectangular distributions with means zero was repeated 1000 times. In sets of three, the distributions had $\sigma = 0.5$, 1.0, 2.0 respectively, and the tenth, $\sigma = 4.0$. If the tendency toward normality and the Monte Carlo simualtion were both perfect

$$\sigma = \sqrt{3(0.5^2 + 1.0^2 + 2.0^2) + 4^2}$$

$$= 5.585.$$

The average S for 1000 trials was S = 5.671. The results are shown in figure 23. The bell shape of the normal distribution is apparent. A goodness-of-fit test could not reject normality at the 90% level of confidence.

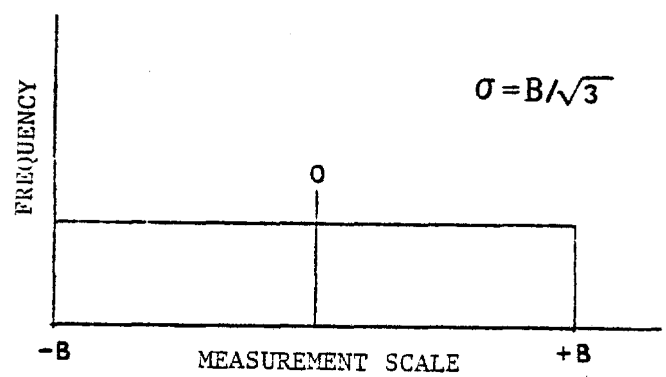


Figure 22 — The assumed frequency rectangular distribution of the systematic error (β) as a function of the limit B.

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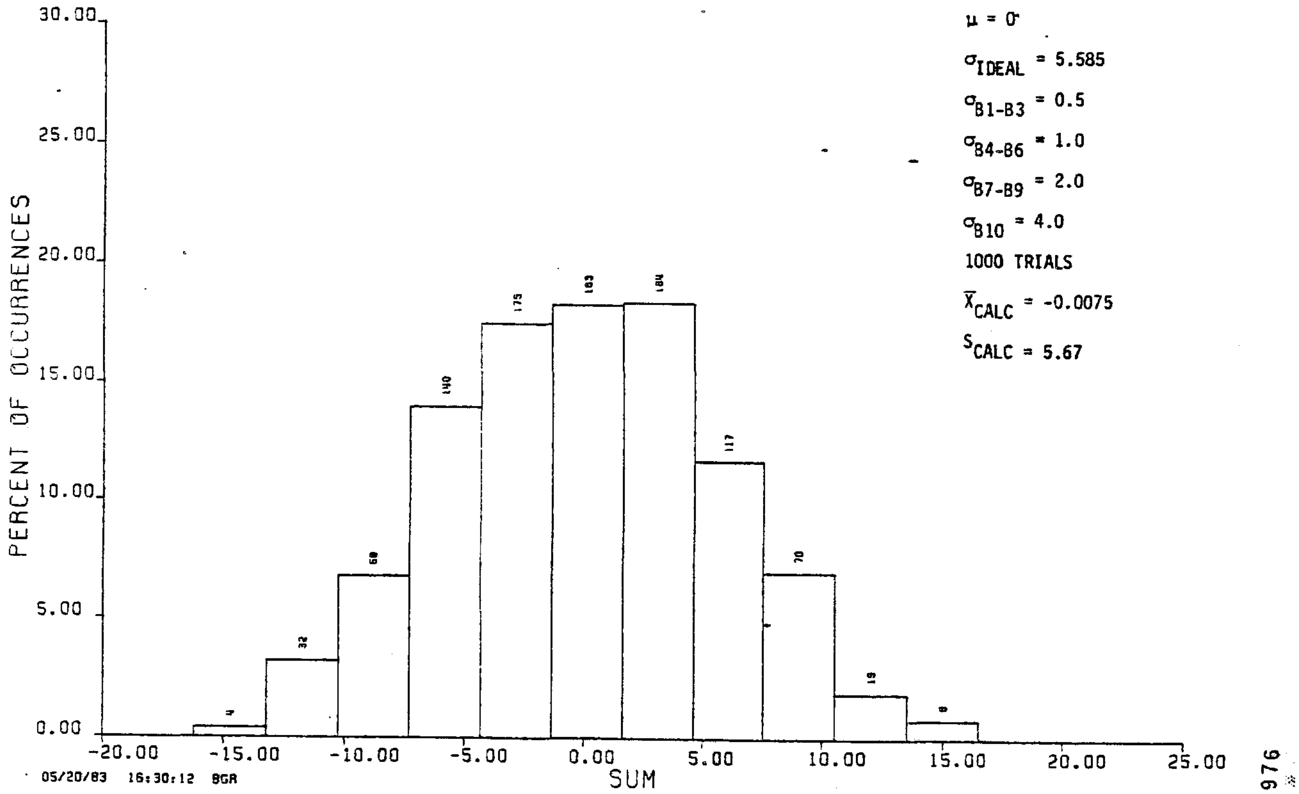

Figure 23 — Distribution of sum of 10 rectangular systematic errors

Annex F: Uncertainty interval coverage

Introduction

A rigorous calculation of confidence level or the coverage of the true value by the interval is not possible because the distributions of systematic error limits, based on judgement, cannot be rigorously defined. Monte Carlo simulation of the intervals can provide approximate coverage* based on assuming various systematic error limits.

F.1 Simulation results

As the actual systematic error and systematic error limit distributions will probably never be known, the simulation studies were based on a range of assumptions. The result of these studies comparing the two intervals are:

a) U_{99} averages approximately 99.1% coverage while U_{95} provides 95.0% based on systematic error limits assumed to be 95%.

   For 99.7% systematic error limits, U_{99} averages 99.7% coverage and U_{95}, 97.5%.

b) The ratio of the average U_{99} interval size to U_{95} interval size is 1.35:1.

c) If the systematic error is negligible, both intervals provide a 95% statistical confidence (coverage).

d) If the random error is negligible, both intervals provide 95% or 99.7% depending on the assumed systematic error limit size.

* Coverage as used herein is the proportion of Monte Carlo trials where the measurement uncertainty interval contains the true value.

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Assumptions and Simulation Cases Considered

(1) From 3 to 10 error sources, both systematic and random

(2) Systematic errors distributed both normally and rectangularly

(3) Random error distributed normally

(4) Systematic error limits at both 95% and 99.7% for both the normal and the rectangular distributions

(5) Sample standard deviations based on sample sizes from 3 to 30

(6) Ratio of random to systematic errors at 1/2, 1.0 and 2.0.

Table 17 — Uncertainty intervals defined by non-symmetrical systematic error limits

B^-	B^+	$t_{95}S_x$	U_{99}^- (Lower limit for U)	U_{99}^+ (Upper limit for U)
0 deg K	+10 deg K	2 deg K	−2 deg K	+12 deg K
−3 Kg	+13 Kg	2 lb	−7 Kg	+17 Kg
0 P_a	+7 P_a	2 P_a	−2 P_a	+9 P_a
−8 deg K	0 deg K	2 deg K	−10 deg K	+2 deg K

F.2 Non-symmetrical interval

If there is a non-symmetrical systematic error limit, the uncertainty (U) is no longer symmetrical about the measurement. The interval is defined by the upper limit of the systematic error interval $(B)^+$. The lower limit is defined by the lower limit of the systematic error interval $(B)^-$. (see clause 7.3)

Figure 24 shows the uncertainty (U) for non-symmetrical systematic error limits. (See table 17.)

$$U^+ = B^+ + t_{95}S \tag{100}$$

$$U^- = B^- - t_{95}S \tag{101}$$

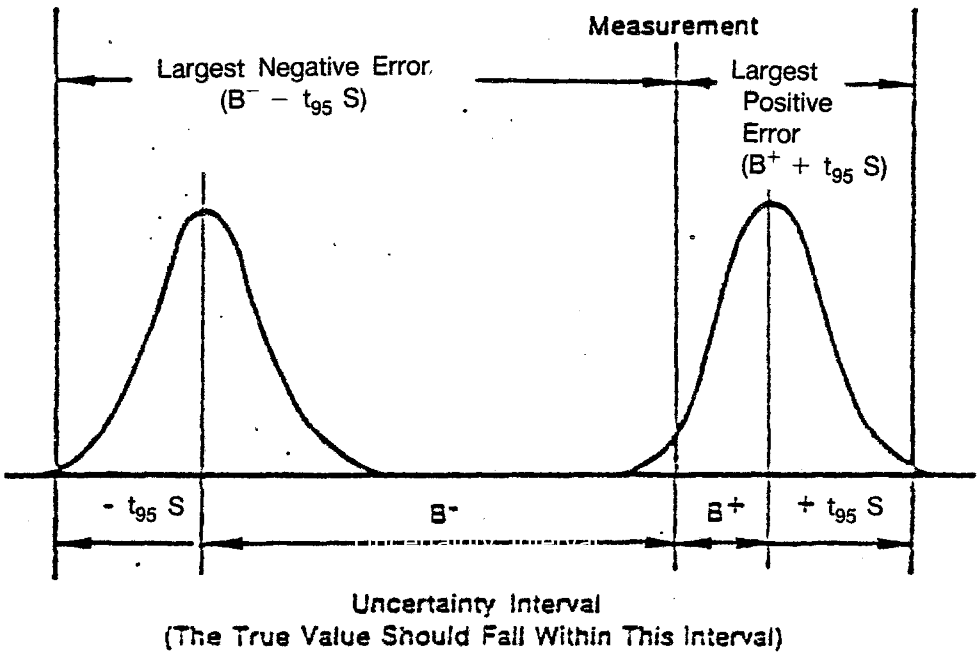

Figure 24 — Measurement uncertainty; non-symmetrical systematic error